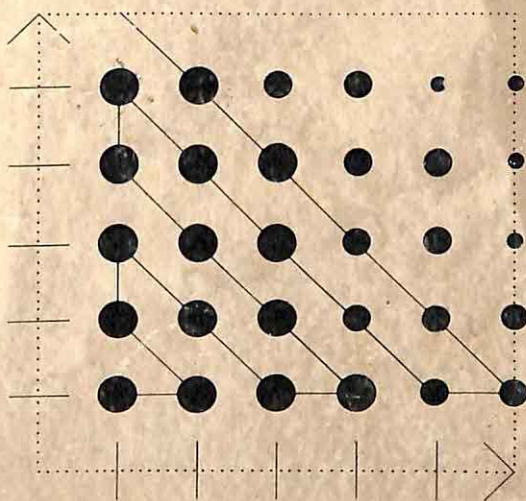


Studies in mathematics education

Volume 1

Edited by Robert Morris

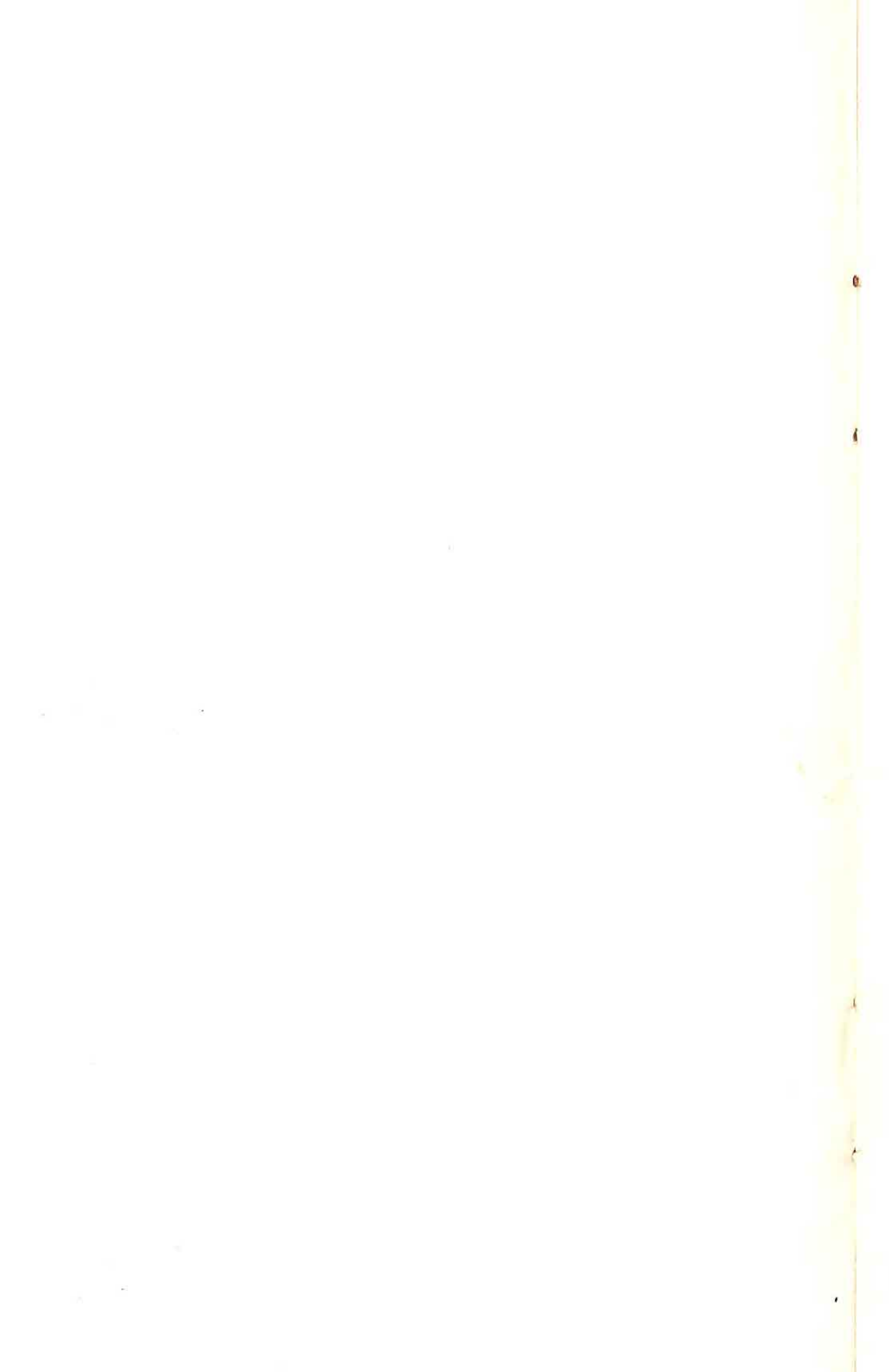
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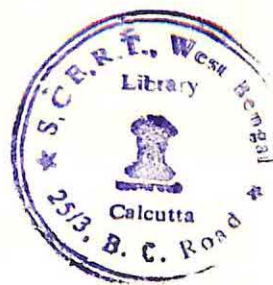
The teaching of basic sciences



Studies in Mathematics Education

Volume 1

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S.C.E.R.T., West Bengal

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(Mathematics - Study
and teaching)

Bureau of Ednl. & Psyl. Research (S. C. E. R. T.)
Date
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Published in 1980 by the United Nations
Educational, Scientific and Cultural Organization
7 Place de Fontenoy, 75700 Paris
Composed by Solent Typesetting, Otterbourne, United Kingdom
Printed by Imprimerie de la Manutention, Mayenne

ISBN 92-3-101779-9
French edition: 92-3-201779-2

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Printed in France

Preface

Unesco has for several years now been involved in pioneering efforts to improve mathematics education in developing countries, going back to 1966 with the initiation of a pilot project in mathematics for the secondary level in the Arab States. That same year, as part of the programme to promote the exchange, at a global level, of information on mathematics education, the series "New Trends in Mathematics Teaching" was established, with subsequent volumes appearing in 1970, 1972 and 1978. The first two volumes contained key articles in mathematics education, whereas the last two were analyses of trends, written by specialists, and scrutinized by international panels of mathematics educators and discussed at international meetings — the last being the International Congress on Mathematical Education held in Karlsruhe, Federal Republic of Germany, in August 1976.

Since obviously trends do not change all that frequently, it was decided to introduce '*Studies in Mathematics Education*' devoted to a single topic. The present publication is the first in this new series. It examines seven specific programmes for improving mathematics teaching in the schools of Hungary, Indonesia, Japan, the Philippines, the U.S.S.R., the United Kingdom and the United Republic of Tanzania, with the aim of making available to a wide public the conclusions drawn from the implementation of these programmes.

These conclusions have not been compared or analysed; rather each paper speaks for itself. The studies reflect some of the reactions of society to the changes in mathematics teaching during the past two decades and their impact.

To edit this study, Unesco sought a mathematics educator with experience in many parts of the world, who could mould the many contributions into a unified volume, while respecting each author's individual style. Robert Morris met these requirements. As former director of the curriculum division of the Centre for Educational Development Overseas, London, he was involved in mathematics projects in many countries before becoming Director of the Schools

Council's Continuing Mathematics Project at the University of Sussex (which is described in this Study). He has frequently been asked to make external evaluations of mathematics and science education programmes in Latin America, Africa and Asia.

By describing the changes in mathematics education in their countries, the authors have succeeded in producing a volume likely to be of interest and practical value to all those who today participate in shaping the policy in that field. Unesco wishes to express its appreciation to Robert Morris and the various contributors to this first *Studies in Mathematics Education*. The articles appearing in this volume express the views of their authors and not necessarily those of Unesco or the Editor.

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Introduction

This first issue of '*Studies in Mathematics Education*' is exclusively concerned with mathematics in secondary education. It is a collection of seven articles, from seven different countries. Six of the articles were specially commissioned. The fourth article – the account of recent developments in the Philippines – is a country report, prepared by the South East Asia Mathematical Society. There is one other exception – 'The Continuing Mathematics Project'. This article deals with one particular problem (innumeracy) at one particular level (in the 16 to 19 age group) in an unusual way (using a multi-media strategy). The other six articles describe developments in mathematics as a part of general secondary education.

Despite their unison of theme, the six general articles are fascinatingly different. They reflect differences in historical context, differences in economic development, differences in social involvement, differences in teaching power and – perhaps most interesting of all – differences in maturity of perspective on the place and treatment of 'modern' topics. Thus, as we read in the third article, Japan is currently in the throes of reform, forced upon the system by dint of democratic demand. Here, the association in the public mind of growth in economic development with extension of schooling has created unprecedented pressure for educational opportunity, and the removal of all obstacles to preferment. So the difference between the content which is 'required' and that which is 'selective' must be abandoned. Differential curricula must go. The 'modern' elements are having to be modified, particularly those which are alleged to be responsible for a decline in students' ability to compute. Examinations, likewise, are having to change, and a new system of selection for university entrance is being instituted.

Should there be any doubt about the absorptive capacity of educational systems, the article from the United Republic of Tanzania is required reading. Here, the legacy of history is an amalgam, combining the contributions of the Arabic coastal trader, the German colonizer, the zeal of the missionary, the mandate of the British care-

taker, the contemporary influence of the multinational corporations and, of course, the modern movement. Despite this *mélange* – perhaps because of it – the introduction of modern maths has enjoyed the strong support of teachers in the tertiary stage, and the backing of Government, where the new ideas are seen as key components of Education for Self-Reliance. But change has been characterized by deep concern for the child. For, as Geoffrey Mmari points out, ‘a goal which emphasises understanding through rigorous proof can be counter-productive if it fails to take account of the ability of the learners’.

In the article from the Union of Soviet Socialist Republics, we receive a résumé of response to the demands of a modern economy, in which mathematics is playing an ever widening role, in agriculture, in commerce, in industry and in technological development. As the article makes clear, the needs are a combination of contrary requirements. On the one hand, there is a need to simplify the mathematics course – to provide that which is most likely to remain part of the currency of mathematics in a technological society, where innovations will have to be absorbed continuously. On the other hand, there is a persistent need to renovate – to keep up with developments in science and in mathematics itself, to cope with the growing complexity of computer technology, and to forge links between mathematics and the maths-using subjects, which are ever more numerous. With these needs in mind, the article interestingly suggests eight criteria which might be used to sort mathematical material. These are: instructional significance; activity; completeness; world outlook; generality; applicability; breadth; and development. Thus a concept such as that of the derivative ranks high for inclusion in any secondary mathematics course since it satisfies nearly all of the eight criteria. Concepts, however, are not all. As the article emphasizes repeatedly, ‘enough time must be allowed for solving problems’.

Recent reforms in Hungary have similarly derived from considerations other than purely mathematical ones. Thus information theory and probability have, for instance, justified a place in the secondary course for their applicability to other disciplines. Other considerations have been the power of certain topics to integrate subject matter. Here are cited sets, transformations, co-ordinate systems and binary operations in arithmetic. Psychological considerations are used to justify a spiral development and an emphasis upon teaching the techniques which apply to the solving of problems. Sociological considerations have influenced both content and method in circumstances where extended educational opportunity has brought in its wake a need to cater for a wider band of ability than in the past.

The Hungarian reforms are also remarkable for their scope, involving, as they have done, the four senior primary grades as well as the whole

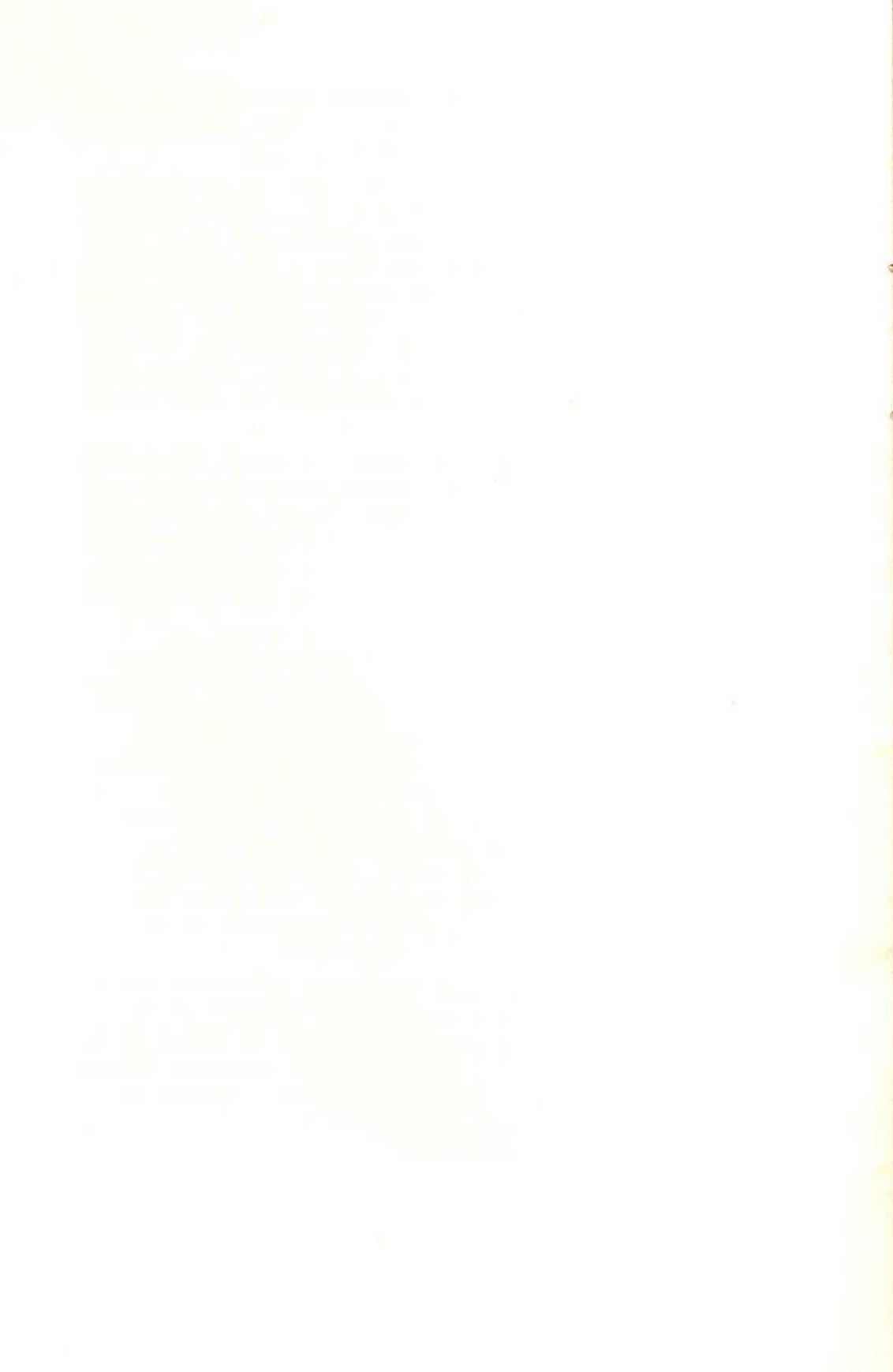
of the mathematics curriculum of the secondary schools. This has called for a huge investment in in-service education for teachers and the deployment of the national network of TV broadcasting.

The modular instructional system, which is currently in the process of being developed in Indonesia, is of special interest for two reasons. In the first place, it is part of a much wider-ranging reform of the curriculum as a whole. In the second place, it has adopted as its principle method the method of self-instruction. This it justifies on the grounds of need: the need to involve learners actively in the process of learning; the need to take account of differential rates of learning; the need to exploit the multi-media; the need to provide the learner with immediate feedback on the result of his efforts to achieve; and the need to ensure 'mastery learning'.

As part of the Development School Project in Indonesia, the modular instructional system in mathematics is seeking, in common with similar developments in social studies, Indonesian language, English language and moral education, to set a new pattern for secondary education by which the system will become more attuned to national goals, to the economic conditions of the country and to the socio-cultural composition of the community.

The account of contemporary mathematical activity in the Philippines is, in effect, an account of a crash programme to cope with shortcomings which urgently require redress. As the article makes clear, the diagnosis of need has been a cooperative venture involving the professional associations for mathematics and those agencies of government which are responsible for the welfare of the subject. It has involved analysis at all levels: at the level of elementary schooling; at the level of senior high school and at the college level. Its scope has embraced a survey of content, of method, of teacher qualification and of teacher preparation. Indeed the main interest in the article from the Philippines is its description of an exemplary approach to the question: what is the mathematical condition of this country?

From this very brief survey of the contents of this issue of *'Studies in Mathematics Education'* it is apparent that the respective authors have succeeded in bringing a remarkable diversity of interest to the unity of its central theme. It is to be hoped that the reader will also detect an engaging harmony of interest in the preoccupations of the many educators who are currently involved in seeking to improve mathematics in secondary education.



Secondary School Mathematics in Hungary

The Hungarian school system

More than 80 per cent of the 3 to 6-year-old children attend nursery school. Those who do not, may go, after they are five, to a one-year preparatory course to ensure that they are as well prepared for compulsory schooling as the others. The preparatory courses are not compulsory, nor are nursery schools. The nursery schools provide educationally rich surroundings, in which mathematics plays a part, especially since the recent introduction of the new nursery school curriculum. Nursery school teachers receive their training either in a four-year institute following eight years of elementary education, or in a two-year tertiary course following matriculation (four years secondary as well as eight years primary education).

At the age of 6, after being tested for school, children begin compulsory elementary school. Those who are not yet ready for school are given a one-year postponement. They either return to a nursery school, or attend school preparatory courses. About 96 per cent of the 6-year-olds begin elementary school. Of the 7-year-olds, about 99 per cent attend school. Over 90 per cent of those who start elementary schooling complete the eight grades.

In the first four years, almost every subject is taught by the same teacher. Until 1961, pre-service training for teaching was given in secondary schools. Since then, it has been given in three-year teacher-training institutes, following matriculation. The subject teachers of grades 5 to 8 are trained in four-year teacher-training colleges (also after matriculation). Teachers of mathematics usually teach another subject, frequently physics or chemistry.

By law, schooling must continue until the age of 16. This means that those who finish elementary school before that age and do not go on to secondary school have to take part in so-called extension training until they become 16. But the majority of those who finish elementary school — in fact more than 80 per cent of an age group — enter either a three- or four-year vocational school, or a four-year secondary grammar school.

The *secondary grammar schools* are attended mostly by those who aspire to a university or college education, and by those who are undecided about their future after completing elementary education. The goal of these schools is matriculation.

The *four-year vocational schools* provide training for medium level specialists of trades which demand some theoretical knowledge. These schools also prepare pupils for college or university education, and terminate with matriculation.

The *three-year vocational schools* provide training for skilled workers who do not require a theoretical background. Their leavers receive a certificate which does not rank as matriculation. They may, however, proceed to a secondary school which prepares for matriculation.

In the vocational schools, the majority of the teachers hold diplomas from five-year universities.

Mathematics is, of course, taught in all the schools so far described. In what follows, however, a detailed description will be given of mathematics teaching only in grades 5 to 8. This is, in part, a consequence of lack of space. It is, however, in these two areas that the curricular and organizational changes have been the greatest, and where consequentially, the focus of interest in mathematics teaching in Hungary currently resides.

The history of the new mathematics curriculum for grades I to VIII

In the summer of 1962, at a two-week international symposium held in Budapest and supported by Unesco, the participating countries (Australia, Belgium, Bulgaria, Czechoslovakia, Denmark, France, Holland, Hungary, Italy, Japan, Poland, Romania, Sweden, Switzerland, the Union of Soviet Socialist Republics, the United Kingdom of Great Britain and Northern Ireland, and the United States of America) unanimously adopted a set of conclusions and recommendations concerning mathematics education. On the basis of these, experiments began in Hungary on a large scale in 1963. These experiments led to more definite formulations of the recommended curriculum and its associated methodological ideas, and, often at the request of teachers, more and more schools took part in them. Eventually a pattern of wider implementation took shape. The experimental curriculum was initiated in the first grade (age 6), and was subsequently followed throughout the first eight years of study. Eventually, in 1974, the Ministry of Education, recognizing the success of the experimental curriculum, adopted the content and the methodology as the definitive

curriculum for the entire country. Its adoption is being effected gradually by steadily increasing the number of classes which utilize the new curriculum. Thus, for instance, during the 1978/79 school year about 7 per cent of the fifth graders and 15 per cent of the fourth graders followed the new curriculum. At the same time, all first grade pupils were being taught according to the new curriculum. This group will complete the eighth grade in 1986, and will matriculate in 1990, though it must be borne in mind that the extension of the curriculum from grades 1 to 8 to the higher grades is still in an experimental stage.

Even though the process of full adoption will take nearly three decades, and may therefore seem to impatient observers to be too slow, it may still be accounted rapid for two reasons. On the one hand, the new course involves significant changes both in subject matter and methodology. On the other hand, the pedagogical preparation of the teachers makes great demands on the higher institutions. Moreover, it is the hope that these three decades will produce a new generation of teachers capable of continuing the reform of the teaching of mathematics even more effectively after the year 2000.

The former curriculum for grades V to VIII

Until 1973/74 the curriculum followed traditional lines similar to that of other countries. Such differences as there were may be explained by the fact that the programme for grades 5 to 8 (devised in 1946 as part of the curriculum for all elementary schools) could not offer all the pupils as much as the old eight-year grammar schools (for selected 10 to 18-year-olds) had provided for the privileged minority. Moreover, with the introduction of eight years of primary education for all, the number of teachers required for grades 5 to 8 had to be greatly increased. This necessitated a rapid, and sometimes less than satisfactory preparation for teaching. As a result the curriculum for grades 5 to 8 became somewhat shallow, but, in view of the realities, this could not be helped. For example, algebra was not taught until the eighth grade, and even there it was limited to the solution of linear equations with one unknown. Consequently, those who completed only eight grades never encountered inequalities or systems of linear equations. The introduction of negative numbers and of powers had also formerly belonged to the eighth grade curriculum. As a result, many such basic concepts had to be taught (under the disguise of 'review') in the grammar schools, whose job of preparing students for matriculation became thereby very difficult. The curriculum of these schools also came under pressure from the increasing demands of higher education upon students of mathematics.

The main features of the curriculum reform

The pressure exerted by higher education on the grammar schools, and transferred by them to the elementary schools, could have made natural a curriculum reform which, in the language of geometry, might have been characterized by the notion of shrinking. In view of the fact that both the preparation of the teachers and the social surroundings of the pupils had improved greatly, it would not have been unreasonable to squeeze the subject matter of eight years into seven, or even six years, thereby evening out the different pressures. However, in the light of the conclusions of the Unesco symposium and of our own experiments, a different solution was chosen. And the resulting curriculum was significantly divergent from the previous curriculum both in its content and construction as well as in its methodology. In fact, it was observed that several other viewpoints had to be kept in mind, and these necessitated a more complete revision of the entire mathematics curriculum.

In what follows, these other aspects will be discussed in relation to the development of mathematics and its application.

Mathematical aspects

Several newly developed areas of study (e.g. information theory, probability and related disciplines) have widened the applications of mathematics. Even if these are only minimally included in the curriculum, it becomes possible to give the pupils a more realistic picture of the power of mathematical analysis.

There is also in contemporary mathematics a trend towards integration. This may be exemplified in the curriculum by avoiding the segregation of geometry into a separate subject. The co-ordinate system is utilized in grades 5 to 8 (and even in grades 1 to 4) to connect algebraic concepts (concepts of number, ordered pairs of numbers, equations, and inequalities) with geometric ones. This continues in the study of functions. Emphasis is also placed upon transformations in geometry. These provide yet another connection between geometrical concepts and numerical concepts, including the binary operations of arithmetic. Thirdly, geometrical shapes are viewed as sets of points, and they, in turn, are viewed as elements of other sets. Thus, for examples, every quadrilateral is a member of the set of all quadrilaterals, while the latter is a subset of the set of polygons. Furthermore, in curricula which reflect a contemporary view of mathematics, geometry has lost its singular role as the major exemplar of deductive reasoning. Instead, a larger role is given to the inductive approach, while, at the same time, and according to the maturity of the pupils, deductive reasoning has been assigned a place in other branches of mathematics.

Even though the inclusion of newer applications has led to an expansion of the subject matter, it is expected that the more unified approach to mathematics will compensate in the use of study time.

Psychological aspects

Mathematics may be regarded as a system of abstract ideas and relationships which recognizably recur within what at first seem to be unrelated notions. This repetition effectively assists the learning of how to solve problems, including solving by mathematical means problems which emerge in disciplines other than mathematics itself. The systematizing of ideas and the solving of problems have a dual relationship. Not only is problem solving assisted by the effective systematizing of concepts, but problem solving techniques contribute considerably to conceptualizing (e.g. the mathematization of open problem situations, and the analysis of various approaches and the resulting solutions). The latest breakthroughs in psychology can better enable us to develop abilities both in conceptualizing and in problem solving. They also direct our attention to questions related to the motivation of learning and, in general, to the fact that the cognitive growth of pupils is but one side of the development of their complete personalities. These considerations were kept in mind in the planning of the study materials and even more so in the formulation of learning methods.

Pedagogical aspects

Even though a number of teachers developed extremely effective methods in their own classrooms (and such methods were adopted by others), the teaching of mathematics, generally speaking, may be more truly characterized as a rigid continuation of old-fashioned methods than as a desire for continuous improvement. It was part of the experimental work to give publicity to worthy traditions, to expand the repertoire of methods of teaching and learning, and to spread the team-work approach and other means of learning and teaching based upon the recommendations of the students themselves.

Sociological aspects

During the past three decades the sociological makeup of Hungary's population has changed greatly. Huge discrepancies in income have disappeared, and the opportunity for learning no longer depends, as it did in the past, upon the financial and social circumstances of the learners. Some pupils, however, are handicapped by less fortunate surroundings (e.g. urban *v.* rural), by their parents' lower education level, their poorer economic situation or other causes (e.g. broken homes). Such differences are magnified in the learning of mathematics by the fact that, due to its strongly hierarchical structuring, it is the

subject in which it is most difficult to fill in the gaps in one's background and to counteract the danger of falling behind, and so becoming discouraged.

In addition, there are, among the general public, deeply rooted misconceptions about mathematics, along with attached fears. The opinion is also prevalent that the learning of mathematics requires special abilities, that the vast majority of people can learn, at most, the formal parts of mathematics, being totally incapable of really understanding mathematical ideas. Regrettably, these attitudes are shared by some mathematics teachers as well. It is believed that the traditional teaching of school mathematics was largely responsible for so shaping the attitude of the public. The manner in which mathematics used to be taught was conducive to the belief that mathematics is separated from the real world, that it exists for itself and that it can be mastered only by a very few. This belief was further strengthened by the introduction of 'the new math' in many parts of the world. This innovation when poorly conceived, made mathematics even less approachable. It further increased its selective function (through pupil failure) and lessened the chances of mathematics being appreciated by those who were initially less inclined.

Bearing these difficulties in mind, an attempt was made to formulate a curriculum (and a corresponding methodology) which makes it possible to motivate each pupil according to his abilities, and to teach pupils how a mathematical model of a practical problem can be made and so dealt with that the original problem can be solved. In summary the major aim is the mathematical education of the masses, the development of their abilities, the eradication of mathematical illiteracy and enabling everyone to master mathematics in accordance with his abilities.

The structure of the curriculum

Although familiar with ways of providing mathematics education which do not depend upon the age of the pupils, it was decided, for various reasons, to retain a breakdown of the curriculum according to grade level. The aim was to make it adaptable to the needs of every school in every part of the country, and every pupil in every school. In addition, an effort was made to ensure sufficient time for the underlying concepts to be gradually assimilated, so that teachers and pupils would have adequate preparation for new topics. These, and the other considerations mentioned above, led to the following structuring of the subject matter.

In all, several hundred concepts are encountered in grades 5 to 8. These, however, can be organized into a dozen or so bundles as follows:

- Sets and logic
- Relations, functions, sequences;
- Arithmetic, algebra;
- Geometry, measuring;
- Combinatorials, probability, statistics.

These topics, interrelated each with another, reappear year after year and together comprise the material for mathematics instruction.

The detailed curriculum, broken down for each year, lists the material to be covered within each topic, and suggests appropriate enrichment material to satisfy the needs of the faster, more interested pupils. Each year, only that smaller portion of the material becomes part of the basic requirement whose concepts can be assumed to be well-understood. Also listed are those concepts and skills previously encountered which need to be mastered because of their relevance to new ideas. Special attention is drawn to the minimal requirements which are necessary for further study. In order to simplify description, if attention is restricted to just one school year, then what has been said above may be illustrated by four concentric circles. Starting with the innermost, they are:

- the minimal requirements;
- the general requirements;
- the subject matter;
- the enriched subject matter.

Within these limits the teacher retains a certain amount of freedom to individualize according to the needs of his pupils.

Curriculum and requirements for the seventh grade

In this section, the material covered in the seventh grade, along with the expected proficiencies, is described in detail in order to exemplify the general description of the previous section. Topics intended to provide enrichment are indicated by vertical lines at the side of the text. Minimal requirements are given in italics. The allocations of time to the various topics is approximate. Each of the available 128 class periods lasts 45 minutes. In the eighth grade, the number of available class periods is the same. In the fifth and sixth grades, it is 160. The material to be studied now follows.

Content

Sets, logic

Flowcharting mathematical and non-mathematical algorithms. Refinement of flowcharts.

The negation of compound (i.e. universally or existentially quantified, conjunctive, disjunctive, etc.) statements in different universes of discourse (e.g. pupils of the class or the school, whole numbers or rational numbers, triangles, quadrilaterals and other geometric shapes or points thereof, etc.).

Investigating the truth values of statements with respect to different universes.

Examples:

- a) All even numbers are greater than 20.
- b) There exists an even number which is greater than 20.
- c) There is no even number smaller than 20.
- d) Among the numbers greater than 20, none are even.

Equivalent reformulation of statements.

Example:

If a number is divisible by 6, then it is even. If a number is not even, then it is not a multiple of 6. All multiples of 6 are even. There is no odd number which could be evenly divided by 6.

Implications, proofs in different areas of mathematics. Reversible and non-reversible implications.

Arithmetic, algebra

Relationships of the type $u = xyz$, $u = xy/z$, etc. in connection with ratios.

Recurring decimals.

Approximating numbers with intervals.

Calculations with approximations. Addition, subtraction, multiplication and division with approximate values. (How many decimals are accurate? How many decimals are worth taking?)

Calculations with rational numbers. Guessing the approximate value of the results. The formulation of algorithms. (Multiplication and division of fractions, the role of the sign in breaking down parentheses, in division and in multiplication.) Generalization: resulting formulae. Multiplication and division of powers with the same

base, or with the same exponent. Powers of powers. Searching for relevant identities. Powers with negative exponents. Normal forms of numbers.

Linear equations, inequalities and systems with one and two unknowns; applications thereof.

Expressing unknowns from equations involving several unknowns.

Graphical illustration of the solution of equations and inequalities in one and two unknowns.

The solution of equations in the form $(x - 2)(x + 3)x = 0$. Solving simple equations such as $x^2 - x = 0$ by factorization.

Relationships between the roots and the coefficients of quadratic equations in which the coefficient of the quadratic term is 1.

Products leading to important identities (e.g. $(a + 1)^2$, $(a + b)^2$, $(a + 1)^3$, $(x - 2)(x + 2)$, etc.

Functions, sequences

Observing the pattern in sequences; expressing the n -th term in terms of n , or in terms of the previous elements of the sequence.

Problems leading to the recognition of arithmetic and geometric progressions.

Problems leading to the summing of the members of infinite geometric sequences.

Examples:

We halve the unit interval AB , halve the resulting subinterval on the right and repeat this procedure endlessly.

$$\begin{array}{c} \text{A} \quad \text{B} \\ \hline \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1 \end{array}$$

For 6 Forints one can buy a slice of chocolate and a coupon. (A coupon comes in the wrapping with each slice of chocolate bought). One can also get a slice of chocolate (with a coupon) for four coupons. What should be the price in Forints of the chocolate without a coupon?

The decimal form of sums of the type

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots ; \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots, \text{ etc.}$$

The comparison of these to the decimal expansion of fractions like $\frac{1}{2}$, $\frac{1}{3}$, etc.

The sketching of functional relations from daily life (time, length, etc.) in the rectangular co-ordinate system.

Examples of quadratic and cubic expressions (areas of squares, volumes of cubes, etc.).

The absolute value as a function; the greatest integer function.

The square root function.

Simple functions constructable from the above ones.

The analysis of functions given by tables, graphs.

Observing the average rate of change in given intervals on the basis of measurements, graphs, tables, formulas, etc.

Geometry, measurements

The Theorem of Pythagoras and its converse. The use of tables for finding squares and square roots. Applications: the length of diagonals, the distance between points in the co-ordinate plane.

Constructions with Euclidean and other tools (right triangle, taut string, etc.).

Simple proofs using reflection. Proof of the fact that the perpendicular bisectors of the sides of triangles are concurrent.

Simple loci. The construction of a circle through three given points. The construction of a circle tangent to three given lines.

Congruence as a distance-preserving transformation (in 2 and 3 dimensions).

Similarity as a ratio-preserving transformation (in 2 and 3 dimensions).

Affine transformations.

Ratios of perimeters, areas, etc. of similar figures.

Ratios of surface areas, volumes, etc. of similar 3-dimensional shapes.

Projections on to the plane from a centre, or with parallel rays.

Example: Projecting an ellipse into a circle.

Polar co-ordinates. Locating points on a sphere.

Combinatorials, probability, statistics

Changing the variables in combinatorial problems, tabulating the

answers, analysing and comparing such tables.

Pascal's triangle and some of its properties.

Dependent and independent events from everyday life.

The multiplication of probabilities.

The application of diagrams in calculating probabilities, including cases when the events do not occur with equal probability.

The calculation of probabilities using tables of random numbers.

An intuitive view of correlations (via diagrams).

Suggested class periods to be devoted to each topic

Work on the basic material –	about 88 periods.
Further examples, enrichment topics –	about 40 periods.
Of which:	
sets, logic –	about 5 per cent;
arithmetic, algebra –	about 40 per cent;
relations, functions, sequences –	about 20 per cent;
geometry, measuring –	about 25 per cent;
combinatorials, probability, statistics –	about 10 per cent.

Expected proficiencies

The pupils should learn to distinguish between an implication and its converse, and be able to decide whether they are true or false. They should be able to use statements involving 'if and only if'. *They should be able to restate, prove and negate statements involving universal and existential quantifiers.*

– *They should be able to perform calculations with rational numbers (positive, negative and zero) given in various forms and given initial estimations for the results.*

– They should be able to set up equations and inequalities corresponding to word problems. *They should be able to solve equations and inequalities, as well as systems, in one or two unknowns by trial and error, algebraically and graphically.*

– They should be able to express the general term (n th term) of simple sequences in terms of n or in terms of the previous members. *They should be able to read tables and graphs and recognize simple relationships.*

– They should be familiar with the Theorem of Pythagoras, and its converse, and be able to apply these in simple situations.

- They should be familiar with basic constructions using Euclidean and other tools. In particular they *should be able to divide a segment into equal parts*, and perform reflections.
- They should know the relationships between the perimeters, the areas and the volumes of similar geometric shapes.
- They should be able to solve simple combinatorial and probability problems, and compare the results with those based on experience.

The secondary grammar schools

The role of the secondary grammar schools is to provide the basics necessary to the study of the humanities and the sciences. Consequently, those who, upon matriculation, cannot enter a university (about 60 per cent) find themselves unfavourably placed for employment, since secondary grammar schools provide no vocational training. (It must be noted, however, that, upon matriculation, students may learn a trade in the course of one or two years of training. Moreover, certain trades, such as those of dental mechanic or cosmetician, can only be learnt in this way). This is one of the reasons why the Ministry of Education recently affected certain changes in the educational structure of secondary grammar schools. According to this decision, the educational aims of secondary grammar schools must be updated and upgraded, and the requirements of practical life must be kept in mind. The new curricula based on this decision will become effective from the 1979/80 school year, on a year-to-year basis.

Current mathematics education in secondary grammar schools

According to the level of mathematics teaching, one may distinguish three types of secondary grammar schools. The vast majority of the pupils go to grammar schools of the general type. A small portion (about 10 per cent) studies in Mathematics I type classes, while a very few students are privileged to continue their studies in Mathematics II type classes. Of the latter, there are only seven in the country, three of them in Budapest. In these classes, the level of teaching approaches university standards. The number of mathematics class periods is very high (usually ten per week), and the pupils are especially talented in mathematics. Pupils who wish to attend these classes have to take an entrance examination in mathematics.

In general, the pupils must decide rather early on the type of school

they wish to attend, and, if they are not successful, the opportunities for changing their course of study are limited. At the same time, those who prove to be talented in certain areas cannot switch to concentrate on them, and, consequently, do not receive appropriate training. This rigidity inherent in the present system was another reason for the structural changes recently brought about.

The facultative education system

The aforementioned organizational changes are embodied in the facultative system, now to be discussed.

During the first year, all pupils take the same courses. This is the 'year of leveling'. As the pupils come from different elementary schools, it is necessary to ensure a uniformity of knowledge, to standardize skills, to upgrade viewpoints, or, at least, to reduce the differences between them.

The second year is the 'year of orientation'. During this stage, the schools assist the pupils in deciding which areas of study they are most interested in and talented for. They are also provided with a certain opportunity to study these subjects more intensively.

The third and fourth years are totally facultative. That is to say, the pupils can elect to devote a larger number of class periods in smaller classes to the course of study they choose. The available courses are not limited to the areas taught in the first two years; they include many others. Opportunities are given for more applied (and applicable) courses of study, assisting those who wish to find a job rather than to continue their studies after matriculation. So, beginning with the third year, pupils can either intensify their studies or prepare for various occupations.

Several secondary grammar schools of Hungary have experimented, and are experimenting, with various models of the facultative system. The Ministry of Education set up a committee which analysed these experiments and recommended the adoption of the system described above. This recommendation was accepted, and, as a result, the facultative system will become the basis for education in Hungary's secondary grammar schools.

Mathematics education within the facultative system

This section will describe the opportunities provided by the facultative system for mathematics education. To begin with, the Mathematics II

type of schools will be only minimally effected by the changes. Their function, since there is an increasing demand for creative mathematicians, remains the same as before: the education of the mathematically super-talented. In other schools, mathematics will be taught on three levels beginning in the third year of studies. In the first two years, the number of class periods devoted to mathematics will be 160 and 128, respectively. In the third and fourth years, the number of class periods is as shown in Table 1.

Table 1. Number of Class Periods in the third and fourth years

Year	3	4
Basic level	96	84
Facultative Level A	160	168
Facultative Level B	224	224

The material included in the basic level is that which is required of all pupils. It forms part of the material of levels A and B as well, even though the three programmes are quite separate from one another, and even the material which is common is covered in separate classes. The basic level is chosen by those students who are least interested in mathematics, and whose chosen fields depend little on mathematical background. Levels A and B are for pupils who are more interested in mathematics, and whose chosen fields of study are highly mathematical (e.g. the natural sciences). The separation of the different levels allows for a deeper discussion of the material which is common to levels A and B. Thus, for example, while at the basic level the calculation of areas, volumes, etc. is done by elementary means, at levels A and B the method of integration is utilized. This difference extends to the choice of problems as well.

It should, however, be noted that, due to a lack of personnel and of material, not every school will immediately be able to offer the facultative level B type courses. Nevertheless, it is expected that their numbers will grow, and that, even at the beginning, there will be more schools with level B curricula than there were schools of the Mathematics I type. Furthermore, the small class size of level A and B type courses may further increase the effectiveness of mathematics education.

The circumstances of introducing the new curricula

In view of the gradual transition to the new curriculum in grades 1 to 8, the position in the grammar schools will be rather complicated until the 1986–87 school year. Until 1981, some of the entering pupils will be prepared according to the old curriculum, others according to the temporary curriculum. During the years 1982–1985, the preparation of pupils will be affected by the temporary and the new curriculum. So, until 1986, the revised curriculum of the secondary grammar schools will have to allow for the various differences in the preparation of the entering students. However, since it is not known what percentage of the entering pupils will have used the two curricula, it is not possible for the secondary grammar schools to cater for change as simply as in grades 1 to 8.

Consequently, for the time being, the structural changes and the desired updating of the material and the methodology will have to be circumscribed by the realities.

The experimental basis for the new curricula

In addition to the experimental work done by the various secondary grammar schools – to be exact, ten schools were involved – assistance was given by the János Bolyai Mathematical Society, which also conducted various experiments in anticipation of the needs of mathematics education after the year 2000. At present, work on this problem is being done under the joint auspices of the János Bolyai Mathematical Society and the Hungarian Academy of Sciences. Several of the findings of these activities have been incorporated into current plans. Heavy reliance has also been placed upon the positive and negative findings of the critical evaluation of the old curriculum, as well as upon many panel discussions, and the views of experts (including sociologists, university professors and others) throughout the country on the new curriculum.

The subject matter of the new curriculum in the secondary grammar schools

As the curricula of the schools are in a state of transition, only a brief description can be given of the mathematical content of the curriculum at the three levels already mentioned.

Basic level

Major topics during the first two years will include: sets of points; sets of numbers; arithmetic; functions; algebra; basic concepts of geometry; symmetry with respect to lines and points; translation; vectors; rotation; basic properties of plane figures; real numbers; the theorem of Pythagoras; square roots; similarity and its applications; trigonometry; transformations by functions; second degree equations; inequalities; combinatorials.

Neither set theory nor mathematical logic were covered in the old curriculum of grades 1 to 8. Both of these facilities the relating of many of the pupils' rich experiences of handling mathematical tools, and thereby pave the way for the introduction of mathematical modelling.

In the study of functions, the pupils become familiar not only with various means of representation, but also with the most important functions and their basic properties.

Algebra is concerned with the properties of operations, equivalent forms, identities, equations, inequalities and systems of equations and inequalities.

The study of geometry is based on transformations. The development is not axiomatic. Basic properties of the various transformations are utilized in deriving the usual theorems of geometry. With respect of each transformation, its invariants are carefully analysed.

The introduction of the real numbers is done intuitively, as the pupils are not yet mature enough for a constructive approach. The basic properties of the real numbers are accepted as axioms, and opportunities are provided for a deeper understanding of their properties (including the relative size of the sets of the rational and of the irrational numbers).

Trigonometry is introduced via the vector co-ordinates, $\sin \alpha$ and $\cos \alpha$. Thus the usual results concerning right triangles appear as theorems. On the basis of experience, this more general approach is preferable to the original.

In the study of combinatorials, the mentality — rather than the formulae — is emphasized. This field thereby provides rich mathematical experiences.

As mentioned above, the three programmes are separated after the first two years.

Topics at the basic level in the third and fourth years

These include exponents, roots, logarithms; applications of trigono-

metry; basic ideas of numerical analysis; co-ordinate geometry; combinatorials; sequences; calculation of perimeters, areas, volumes and surface areas; organized summary.

The first topic covers the basic properties of the exponential and logarithmic functions.

In continuing the study of trigonometry, the scalar product is introduced along with the laws of sine and cosine, basic identities are derived and trigonometric equations are solved.

The basic ideas of numerical analysis are introduced in order to give all pupils a more complete view of a mathematical mentality.

Co-ordinate geometry deals with straight lines, circles and parabolas. Through some simple examples linear programming is also introduced. Beyond the solution of concrete problems, the emphasis is on introducing co-ordinate geometry as one of the major patterns for the solution of mathematical problems.

In the study of combinatorials some basic ideas of graph theory are introduced. Again the emphasis is on methods rather than on facts. The properties of Pascal's triangle are further analysed.

Mathematical induction is introduced via the study of sequences. Special attention is given to arithmetic and geometric progressions.

Within the last topic, the pupils become aware of the need for higher level tools, but, for the time being, limitations of time do not allow for the introduction of analytical methods. This will change when the new curriculum for grades 1 to 8 has become fully operative. Nevertheless, the volume and surface area of several basic shapes are derived, and the pupils are given some ideas about the tools necessary for extending these results.

The organized summary at the end of the fourth year is much more than a review of the topics covered in the course. The review is made from an advanced viewpoint. It includes many problems which can be solved only by gaining complete understanding of the entire material. Many concepts are defined in their full generality only at this stage. The most fundamental questions concerning mathematics are dealt with at this point.

It was mentioned earlier that many difficulties must be resolved during the transition period. To assist in this matter, the teachers will be given more academic freedom. They will, on the one hand, be free to devote more time to the study of the basic proficiencies, should the need arise. On the other hand, they may wish to introduce various concepts differently.

Facultative level A

The main difference between the mathematical content of the basic level and that of level A is that, in the level A programme, the elements of analysis are also introduced. In the third year, the concept of the limit of a sequence and its applications are included together with the limits of real valued functions of real variables and the derivative, and its applications. In the fourth year, the topics covered include the definite integral and its applications to geometry and to problems encountered in physics.

In the development of analysis, the emphasis is upon the sound understanding of basic concepts. The major aim is to introduce the ideas as examples of mathematical modelling, and so to make the pupils familiar with the activities of mathematicians. The time allotted is not sufficient for the discussion of complicated methods.

In the study of combinatorials, several new types of problems are introduced, mainly from graph theory. The basic concepts of probability are discussed. The emphasis is on a gradual strengthening of understanding.

Facultative level B

The additional material in the level B curriculum, as compared to that of level A, is as follows: more depth in analysis (e.g. the second derivative and its applications, infinite series, the convergence of improper integrals), further studies in graph theory (special subgraphs, colouring problems) and in probability (the binomial distribution) and the introduction of basic concepts of linear algebra (matrices, vectors, systems of linear equations and their applications).

Concluding remarks

Both at levels A and B approximately thirty class periods are available for the teacher to use as he thinks appropriate. He may choose to introduce additional topics or to devote more time to increasing the proficiency of his students in solving problems.

The facultative curricula described above will be adopted after two years trial in the ten secondary grammar schools already mentioned. So far, the first year's material has been tested in only one *gymnásium*. Detailed plans have been completed for only the first two years of study. Work is currently in progress on the curriculum for the third and fourth years. The textbooks and related documents will be published by the Educational Publishing House, Tankönyvkiadó Vállalat. The writers were competitively chosen. Through the use of problem sheets and workbooks, the methods of individualized instruction and

discovery will be encouraged. There is an emphasis on more problems of the applied type.

The preparation of teachers for the new curricula

This work will be done in several steps in various ways. Only those teachers who have concluded a preparatory course will be allowed to teach the new elementary school curriculum, while inspectors will themselves be prepared through central courses. This work has been in progress since 1974. It also involves the assistance of the Hungarian Television. So far, some forty programmes (about 1800 minutes of viewing time) on the new curriculum have been completed, and they are repeatedly broadcast throughout the year. Furthermore, the programmes are video-recorded by the National Centre for Educational Technology (00K). Copies are sent to county pedagogical institutes, thus making the programmes utilizable repeatedly, in accordance with local needs. In the period of preparation, a number of motion pictures and other tools were also produced. In this way, mathematics teachers of grades 5 to 8 may be prepared, even if to varying degrees, for this arduous task through organized training and self-instruction.

In view of the smaller degree of changes in content, the preparation of teachers of secondary grammar schools requires no comparable effort. During the school year, teachers are instructed on five or six occasions about the new curriculum, the related material and the possibilities offered by facultation.

Teaching materials

The publication of documents (textbooks, working papers, etc.) is in the hands of Tankönyvkiadó Vállalat. This house announces competitions for new textbooks and other printed material. The entries are ranked by a board, whose decision underlies the choice of the authors to be assigned for the production of textbooks.

Parallel with the introduction of the new curricula, the teachers are to be given more and more methodological freedom. For example, two sets of textbooks for secondary grammar schools are to be published instead of one as earlier. The layout of the first one will be traditional while the other one will deal with the material in line with the aforementioned experiments. The teachers will be free to use whichever textbooks they wish for a given grade.

Other teaching materials for the new curricula will be provided by the Ministry of Education from its approved list of demonstrational and

learning aids appropriate for teaching at both the elementary and the secondary grammar school levels. The acquisition of these aids (motion pictures, transparencies, logic blocks, coloured rods, etc.) has practically been concluded in elementary schools; it is currently in progress in secondary grammar schools.

Soedijarto, R. Ibrahim, and A. Khodir

The Indonesian Modular Instructional System for School Mathematics

Origins of the Project

General Background

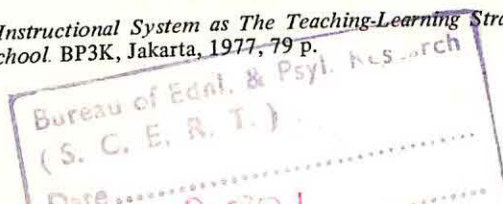
Since the proclamation of independence, in 1945, Indonesia has expressed a commitment to universal schooling, as stated in its Constitution and education law.¹ As a result of this basic policy, the expansion of educational opportunity has reached a stage of development where, by 1977, almost 80 per cent of elementary-school-age children (7–12 years of age) are in school, almost 70 per cent of the elementary-school graduates go to junior high school, and 80 per cent of junior high school graduates go to senior high school.²

Since the beginning of the first Five-Year Development Plan in 1969, Indonesia, while continuing and intensifying its commitment to universal schooling, has realized the importance of improving her educational system, making it more relevant, effective, and efficient, notwithstanding the complexity of the task of managing an education system of a nation of 140 million people scattered among 3000 inhabited islands stretching across 3000 miles of ocean. In pursuit of relevant, effective and efficient education, it has been the belief of educational planners that curriculum design is of paramount importance. For this very reason the Development School Project, a research and development programme to reform the elementary and secondary school system³, has stressed the reform of the system, including its objectives, content, teaching/learning strategies, evaluation procedures, and admini-

1 Undang-Undang Dasar Negara Republik Indonesia 1945 [The Constitution of the Republic of Indonesia 1945]. In: *Himpunan Ketetapan-Ketetapan MPR 1978*, p. 201–25, C.V. Pancuran Tujuh, Jakarta, 1978. 225 p.

2 Badan Penelitian dan Pengembangan Pendidikan dan Kebudayaan Departemen P. dan K. *Rangkuman Statistik Persekolahan Departemen P. dan K.* [The Summary of School Statistics of the Ministry of Education and Culture]. BP3K Departemen P. dan K., Jakarta, 1978, 24 p.

3 Soedijarto. *The Modular Instructional System as The Teaching-Learning Strategy in the Indonesian Development School*. BP3K, Jakarta, 1977, 79 p.



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stration. Within this framework, the key decision was to adopt a modular instructional system as the teaching/learning strategy.

The principal motivating forces

The curriculum of Indonesia's elementary and general secondary schools has three main components: a general education component, designed to develop morally strong citizens, healthy in mind and body; an academic education component, designed to develop intelligent citizens; and a skill education component, designed to develop competent citizens.⁴

Mathematics is one out of the curriculum's four basic academic subjects. The others are science, social studies, and languages. The Development School Pilot Project has so far been concentrating on these four academic subjects, and on skill education. Thus mathematics has, since 1975, become one of the subjects adopting modular instruction as its teaching/learning strategy.

In his article 'The Modular Instructional System as the Teaching/Learning Strategy in the Indonesian Development School', Soedijarto described the rationale of adopting a modular instructional approach.⁵ From the decision to extend this approach to mathematics, three major factors may be adduced: the quality of mathematics education; the quality and quantity of mathematics teachers; and the introduction of new mathematics into Indonesian schools.

The quality of mathematics education

In analysing this factor, planners used two indicators as criteria for evaluation: the student's achievement; and the learning processes experienced by the students.

When the decision to use a modular approach was taken in 1974, there were no hard data about the quality of education in terms of student learning achievement. The decision was therefore based on an informal observation that suggested a decline of student achievement in mathematics. Planners designed a study of the quality of education. The results of this study confirmed the informal observation. Thus, while at the elementary school level, the national mean score in arithmetic has been 50 per cent of the maximum score (ranging from 46

4 Departemen P. dan K. *Kurikulum SMP 1975* [Junior High School Curriculum of 1975], Vol. I. Departemen P. dan K., Jakarta, 1976, 24 p.

5 Soedijarto, *op. cit.*

per cent in one district to 63 per cent in another)⁶, the national mean score at the junior high school level is only 37 per cent of the maximum score (ranging from 24 per cent in one area up to 47 per cent in another)⁷. Although at the senior high school the study has not yet been concluded, tentative results suggest that the quality is at least as poor as that at the junior high school level.

As to the effectiveness of learning, it has long been known that the traditional mode of teaching is far more effective in selecting students with talent than in helping students to develop their talent.⁸ Indeed, it can, in general, be said that, at the end of a typical lesson, only a few students will have mastered most of the work planned by the teacher; the majority of the students will not have mastered more than half of the work, and, among the rest of the students, the slowest learners will not have mastered most of the work. This is because the underlying assumptions of the traditional learning model seem to be: that all students can follow the lesson at the same rate of learning; that all students have the same style of learning; and that all students will be able to finish, and master, the same objectives using the same method at the same time. So there is a tendency for most teachers to be satisfied when only a few top students understand and master the objectives, and the needs of the slowest are neglected. This has cumulative consequences. Both the students from the average group and those from the slowest group decline in their achievement. This can happen for several reasons: through a frequency of failure, the average and slowest students tend to lose confidence in their capability; teachers do not usually have enough time to monitor the progress of all the students and to help them to improve; lessons are usually graded in difficulty, with the earlier ones being a pre-requisite for the mastery of subsequent lessons; teachers are often unable to diagnose learning difficulties or to remedy them, for most teachers do not have either the time or the skill to do so.

For these reasons it is clear that, within the traditional school learning model, most students experience academic disadvantages, and that, by the end of a unit, their academic disadvantages have accumulated. This is especially true for the average and the slowest groups. Thus it is not surprising that the quality of education, at least in terms of learning

6 Moegiadi *et. al.* *Penilaian Nasional Kualitas Pendidikan Di Tingkat Sekolah Dasar* [National Assessment of the Quality of Education at the Elementary School Level]. BP3K Departemen P. dan K., Jakarta, 1976, 96 p.

7 Mangindaan, Christina, *et. al.* *National Assessment of the Quality of Indonesian Education: Survey of Achievement in Grade 9; An Interim Report*. BP3K Departemen P. dan K., 1978, 75 p.

8 Bloom, Benjamin. *Human Characteristics and School Learning*. McGraw-Hill Book Company, New York, 1976, 284 p.

achievement, has been low. Moreover, in the case of mathematics, this has occurred not only in Indonesia, but in each of the twelve member countries of the Organisation for Economic Co-operation and Development (OECD) that participated in the International Study of Achievement in Mathematics.⁹

The quality and quantity of mathematics teachers

It has long been known that to supply qualified mathematics teachers has been one of the most intractable problems faced by educators in Indonesia. A study conducted in 1971 showed that only 21 per cent of those teaching mathematics at the junior-high-school level were professionally trained as mathematics teachers.¹⁰ Since then the situation has not significantly changed.

The introduction of new mathematics into Indonesian schools

As an effort to make the content of mathematics at elementary and secondary levels more relevant to the demands of national development, 'new mathematics' were introduced into Indonesian schools. Introducing new content to professionally trained teachers is not an easy task. It is even more difficult when they have not been professionally trained. For this reason, one of the crucial questions to be answered has been 'How can "new mathematics" be brought into the content of education, using the existing cadre of teachers, without damaging the quality of education for Indonesia's younger generation?' Introducing the system of modular instruction is, in part, a reflection of the effort to answer this question.

Sources of funding

The Development School Project is one of the research and development programmes in the National Five-Year Development Plan. Thus, the budget for developing the modular instructional system in school mathematics, being a segment of the Development School Project, is a part of the government's budget for development. This budget includes financing the cost of the following activities: developing instructional material; trying out instructional material; evaluation and monitoring activities; developing training packages and the training programme itself; developing prototypes of educational equipment and media to

9 Husen, Torsten (ed). *International Study of Achievement in Mathematics: A Comparison of Twelve Countries*, Vol. II. Almqvist & Wiksell, Stockholm, 1967, 368 p.

10 Thomas, R.M., and Winarno Surachmad. *The Quantity and Quality of Teachers in Indonesia, vol. 3: Secondary Schools: The Quantity and Official Qualifications of Teachers*. Office of Educational Research and Development, Jakarta, 1972, 139 p.

support the implementation of the modular plan; other research activities relevant to the Development School Project.

It is also important to mention here that the only external input received by this programme is from the United National Development Programme (UNDP)/Unesco. This has provided both fellowships, study tours, and other training abroad for upgrading personnel involved, and technical assistance in the form of short-term consultants and educational equipment. The total amount of the external input has been less than 10 per cent of the total budget provided by the government.

Goals

One of the basic assumptions which underly the Development School Project is that schooling is a function of the social system of the nation in which the school is operating. For this reason, the curriculum of the Development School Project has been based on educational objectives derived from the educational needs of national development, themselves identified in a special study on the subject.¹¹ Thus, the objectives of the curriculum have been founded on national goals, the socio-cultural setting, and the economic condition of the Indonesian society of which the school is an integral part. Before we discuss school mathematics as an object of learning and the appropriateness of modular instruction as the teaching/learning strategy, we need to look at both the socio-cultural setting and the economic conditions of the country, and to consider the relevance to the national goals of the decision to adopt a modular instructional system.

National goals

The ultimate goal of Indonesian national development is to achieve a just and prosperous society, materially and spiritually based on the *Pancasila*.¹² In as much as there are still gaps between existing conditions and those envisaged in the ultimate goal, The People's Assembly has decided, as a matter of policy, to develop a series of Five-Year Development Plans so that, by the end of the sixth Five-Year Development Plan, the nation will be well prepared to achieve the ultimate goal, as stated in Indonesian Constitution of 1945.

¹¹ Soedijarto, et. al. *Cluster II Identifikasi Tujuan-Tujuan Pendidikan Bagian I: Proses dan Hasil Identifikasi Tujuan-Tujuan Kurikuler* [Cluster II Identification of Educational Objectives. Part I: Process and Output of the Identification of Curricular Objectives]. Badan Pengembangan Pendidikan, Jakarta, 1972, 177 p.

¹² 'Garis Besar Haluan Negara' [State Guideline Policy]. In: *Himpunan Ketetapan-Ketetapan MPR 1978*, p. 73-127. C.V. Pancuran Tujuh, Jakarta, 1978, 225p.

Economic conditions

As the nation progresses through the fourth year of the Second Five-Year Development Plan, the economy is in far better condition than it was in the early sixties. However, it is clear that the economy is still far from the condition it is expected to reach by the year 2000.

Indonesian leaders realize that there are still imbalances to be resolved in the economic sphere if the ultimate target is to be reached. One of Indonesia's leading economists has identified four sorts of basic imbalance: imbalance in productive resources, including human resources and expertise; imbalance in growth among the various sectors of economic activity; imbalance in the distribution of national income; and inconsistencies between the demands of national development on the one hand, and the human attitudes and social institutions on the other.¹³

Socio-cultural conditions

In the realm of socio-cultural conditions, a leading Indonesian anthropologist has identified important aspects of the cultural value system that run counter to the demands of national development. These are: fatalistic attitudes toward life among the majority of people; a lack of appreciation of productive work; a lack of positive attitudes toward the capabilities of man to control nature; a lack of futuristic attitudes; and a lack of achievement-orientated attitudes.¹⁴ Students of socio-cultural development are not surprised to find such attitudes since they are characteristic of the kind of agricultural society in which almost 80 per cent of Indonesians live. It was, for example, observed by Rogers in his book *Modernization Among Peasants* based on his study in Latin America.¹⁵

It is believed by many social scientists that school education can play a decisive role in creating positive attitudes for national development, as identified by Tinbergen as well as by Inkeles.¹⁶ However, it is also acknowledged that not all types of education encourage the attitudes and skills which the development of a modern society requires. Planners

13 Djojohadikusumo, Soemitro. *Indonesia dalam Perkembangan Dunia: Kini dan Masa Datang* [Indonesia in the World Development: Present and Future]. Lembaga Penelitian, Pendidikan dan Penerangan Eko Ekonomi dan Sosial (LP3S), Jakarta, 1977, 229 p.

14 Koentjaraningrat (ed). *Manusia dan Kebudayaan di Indonesia* [Man and Culture in Indonesia]. Jambatan, Jakarta, 1975, 390 p.

15 Rogers, M. Everett. *Modernization Among Peasants: The Impact of Communication*. Holt, Rinehart, and Winston, Inc., New York, 415 p.

16 Inkeles, Alex. The Modernization of Man. In: Myron Wenier (ed). *Modernization*. Basic Books, Inc., 1966.

of the Development School Project are convinced that, unless school learning becomes more relevant, effective, and efficient, it will be impossible for the school to generate these attitudes and skills.

Mathematics

As mentioned above, Indonesia introduced 'new mathematics' into its elementary and secondary school curricula in 1975. Part of the reason for introducing new mathematics as a means of making school education more relevant to national development has been explained by one of Indonesia's mathematicians in terms of the negative features of traditional mathematics. These were: laying too much stress on the memorization of mechanical processes without giving enough attention to developing understanding; a failure to bring out the relationships within mathematics such as those between arithmetic and algebra, and those between algebra and plane geometry, as well as those between mathematics taught in school and mathematics in real life; too much wasted time; too much stress on topics which provide little motivation for students since they fail to appeal to students' scientific curiosity; and too few improvements based on research.¹⁷ From this perspective it is clear that one of the motives for introducing new mathematics into school curricula has been to make mathematics more relevant, more challenging, and more lively.

National Education Objectives

From the previous discussion it is clear that the programme to reform mathematics education is concerned not only to improve the quality of educational achievement and the quality of learning, but also to make mathematics education relevant to the demands of national development. This expectation is in line with the general objectives of Indonesian education. These were stated in the Ministerial decree on the standardization of new curricula for elementary and secondary schools.

'The purpose of the schools is:

To educate a "development-oriented" man with the character of Pancasila, and to develop Indonesians who are healthy, mentally and physically, have knowledge and skills, are creative and responsible, are democratic and tolerant, are able to develop high intellectual abilities, are conscientious, and love the nation and mankind, consistent with the values of the 1945 Constitution.'¹⁸

¹⁷ Kartasmita, B. *Bobot Matematika Lama dan Matematika Modern: Sebuah Tanggapan* [Weight of Old Mathematics and New Mathematics: A Response]. BP3K, a memograph, 1976, 8 p.

¹⁸ Departemen P. dan K. *Kurikulum SMP 1975* [Junior High School Curriculum of 1975], Vol. I. Departemen P. dan K., Jakarta, 1976, 24 p.

How far will the modular instructional system, adopted as the teaching/learning strategy, be able to contribute to the development of a more relevant, effective, and efficient educational system? It is not the intention of this paper to answer this question, but rather to analyse the general characteristics of the modular instructional system, and its function in developing attitudes and skills suitable for modern man.

The fundamental characteristics of the Indonesian modular system

There are seven fundamental characteristics of the modular instruction system. They are: an emphasis on active self-instruction; a concern for individual differences in rates of learning; a clear statement of the learning objectives of each lesson in terms of subsequent student behaviour; the utilization of multi-media and multi-methods; the active participation of the learner in all phases of instruction; the immediate feedback to the learner of evaluation information; and emphasis on mastery learning. Each of these will now be discussed.

The emphasis on self-instruction

In contrast to the traditional mode of learning, in which the pupils' learning is heavily dependent on the teacher's activities, modular instruction stresses the importance of students learning by themselves. The basic assumption underlying this mode is that learning is a process which must be performed by the individual himself as he seeks to develop skills and understandings. The student does the learning. The teacher or textbook or film merely helps in the process.

Thus, a large part of the Indonesian modular system employs the self-learning media in the form of written material that guides students toward the mastery of new skills and knowledge by gradual steps. But since written material is not always the most feasible (as with young children who do not yet read well) or the most suitable (as with students who are learning the skills of making group decisions), the modular system involves the use of other media as well. This will be explained under the fourth characteristic below.

In the modular system the teacher plays a different, more varied professional role than in the traditional classroom. He becomes primarily a manager of class activities, a diagnostician of pupil needs, and a guide of the learning process.¹⁹ Within this model, the teacher has more time than the traditional instructor has to monitor student progress, to diagnose learning difficulties, to help individuals solve their problems, to motivate students, and to carry out other professional jobs

¹⁹ Departemen P. dan K., *op. cit.*

that have rarely been possible within the traditional instructional model.

One of the most important goals of the modular system is to emphasize self-teaching, so that students will develop habits of self-direction, and of reading extensively and enthusiastically. These two habits are considered very important for people of developing nations, since such habits have not typically been part of the indigenous cultures. The Indonesian Minister of Education, Daoed Yoesoef, in speaking of this matter, has stated that the use of written material in Indonesia has neither been intensive nor effective. Yet such use is essential for the development of a modern society.²⁰ For this reason, nurturing a new written tradition in the society should become an integral part of the nation's cultural policy.

The concern for individual differences in rates of learning

In a typical class, there are many individual differences among the learners. It is usually impossible for the teacher to meet all the needs of each pupil simultaneously. So, in a class lesson, the teacher adopts a course which seems best suited to the learning pace of the majority of the pupils. Inevitably, some students find the pace too slow, others too fast.²¹

Self-instructional modules are designed to enable students to learn at their own pace. The slow learner can repeat any part of the lesson he has found difficult. By this means he may eventually master the basic objectives. Meanwhile, the fast learner can move quickly through the programme, and master the objectives sooner than his more average classmates.

The policy of giving students the chance of mastering objectives in accordance with their own rates of progress is based on the assumption that the vast majority can master the objectives if they have enough time to do so.²²

20 Daoed Yoesoef. Pengembangan Kebudayaan dan Kaitannya dengan Pendidikan [Cultural Development and its Relation to Education]. In: *Penjelasan Menteri Pendidikan dan Kebudayaan dan Menteri Muda Urusan Pemuda tentang Kebijakan Pendidikan dan Kebudayaan serta Pembinaan Generasi Muda* [Statement of the Minister of Education and Culture and the Junior Minister of Youth], p. 439. Departemen P. and K., 1978. (Dalam Rapat Kerja dengan DPR.)

21 Russell, James D. *Modular Instruction: A Guide to the Design, Selection, Utilization and Evaluation of Modular Materials*. Burgess Publishing Company, Minneapolis, Minnesota, 1974, 142 p.

22 Carroll, J.B. A Model of School Learning. *Teachers College Record*, vol. 64, 1963, p. 723-33.

The statement of objectives in each module

In the traditional school, the specific objectives of the pupils' learning are often very unclear. As a result, teachers have too often neglected to prepare a systematic teaching/learning programme of logical steps which would carry the learners to their goals. Consequently, both teachers and learners have engaged in many activities that are actually irrelevant to the overall objectives of the school. Such a system is wasteful, and one which developing nations can ill afford.

In an effort to avoid such waste, designers of the modular system have insisted that each module should spell out the specific learning objectives in terms of what the learner should know and should be able to do by the end of the module. Because of this insistence, teachers and module writers are obliged to plan learning activities which are truly relevant to the objectives of each module. The student, moreover, is aided in his learning tasks by seeing exactly what is expected of him. In this way, his work becomes goal-oriented and achievement-oriented, two characteristics we consider important for the progress of a developing nation.

The utilization of multi-media and multi-methods

One of the main objectives of introducing the modular instructional system is to give students a chance to pursue well designed, relevant learning activities. Thus, learning by reading text materials is only one of a broad array of approaches. Others include observing demonstrations, conducting experiments, following a teacher's explanation of difficult concepts, solving problems, watching slides and films, conducting small-group discussions as well as large-group discussions, and listening to tapes. The choice of method is that one which is the most effective means of learning, given the desired objective. So, by adopting a multi-media policy, the designers of the modular system are obliged to develop a range of media and educational equipment. And one factor that must be included in the designers' considerations is the economic feasibility of the media.

The active participation of the learner

Although this matter of active learning has already been touched upon, it is of such importance that it warrants additional emphasis. One of the key features of educational reform in Indonesia is this stress on the student's actively pursuing knowledge by constant problem solving, by answering questions, and the like, in contrast to the traditional classroom in which pupils spend the major part of their time passively observing the teacher perform in front of the class.

The immediate feedback of evaluation information

Besides having clearly stated instructional objectives, each module incorporates worksheets, or work assignments, and evaluation instruments. As a result, each student can immediately check up on his own progress at every step of the learning programme.²³ In other words, when a pupil completes a stage in the learning sequence, he has an opportunity for self-evaluation. And, on the basis of that evaluation, he knows whether to go on to the next stage or whether to review and practise the previously taught skill before moving ahead. This continuous and immediate evaluation is designed to reinforce the pupil's motivation and working habits.

Mastery learning strategy

A key feature of the modular instructional system is a requirement that the student should master the objectives to a pre-specified criterion level before he proceeds to the next module in the sequence. This feature is designed to minimize the number of students who fail to achieve the learning goals, and to ensure that the students adequately understand the material of each module before proceeding to the next one.

In the modular system, as in all other instructional systems, the brighter learners master the objectives before their more average and slower classmates, while the least adequate learners lag behind the rest of the class. However, in the modular system, the slow learners are not left simply to drag along as best they can without special attention. Rather, they are expected to master the objectives of the current module before they move ahead to the next. Within such a system, it is necessary for the faster students to undertake enrichment activities; to expand their learning while the slower ones are still mastering the basic objectives of the current module.

In the Development School's system, 'mastery' is defined as a pupil's accomplishment when he has responded successfully to at least 75 per cent of the evaluation items required in the final assessment at the end of the module. This standard of 75 per cent differs from that of Bloom, who assumes that 90 per cent of the students can master almost 100 per cent of the items.²⁴ There are several reasons for adopting a less stringent criterion than Bloom. In the first place, it is not easy to develop a curriculum of minimum-essential learnings that all students can and should achieve. Designers of the Indonesian system are still

²³ Soedijarto, *op. cit.*

²⁴ Bloom, Benjamin S. *Mastery Learning*. In: Block, J.H. (ed). *Mastery Learning: Theory and Practice*, p. 47-63. Holt, Rinehart and Winston, Inc., New York, 1971, 152 p.

not sure that they have identified the array of objectives that comprise the minimum essentials. Secondly, the modules which have so far been written are still in an imperfect, developmental state. And, thirdly, the criterion-referenced assessment instruments planned for the end of modules are also in a developmental stage. So it would seem more reasonable, in the present state of the Project, to impose the less demanding 75 per cent mastery level, rather than the 90 per cent level appropriate to a well-tryed system.

Hitherto, the designers of the modular system have been concentrating upon developing basic modules without regard for individual differences which may suggest the need for different routes to a particular goal. Now, however, the basic modules have been sufficiently developed to permit the programme planners to direct their concerns to three additional features: the preparation of pre-assessment tests to diagnose the readiness of a student to move into the next module; the development of remedial techniques to help the slower or the confused learner with his learning difficulties; and the preparation of a wide array of enrichment activities to serve the more advanced learning needs of the faster learner, who completes the basic module ahead of his classmates.

In conclusion, Indonesian curriculum planners hope that the Development School's modular instructional system, with the seven characteristics outlined above, will significantly augment those human attitudes and skills which the demands of national development call for.

The Development and Operation of the Project

Content and Scope

As noted already, the curriculum of the Development School has included the introduction of 'new mathematics'. This has not required the total revision of the subject matter that had been taught under the earlier programme. Indeed, the substance of mathematics and arithmetic remains much as it used to be, but there is in the new programme a different stress, particularly in such matters as the instructional approach and the use of terminology. In the traditional curriculum, a great deal of emphasis was placed on calculations, on problems involving the application of rules the students had memorized, and on tricks of calculation that could be employed to find particular solutions.

In contrast, the new programme has been designed to overcome these weaknesses through more emphasis on concepts and structures. Within

the framework of the new programme, mathematics is no longer viewed as a finally determined, or 'dead' subject that permits no further creative thinking. For example, arithmetic is not considered merely a tool, but rather as a way of thinking within a broader framework of mathematical structures. The rationale is that well developed, well understood mathematical concepts enable the student to adapt more readily in a rapidly changing world. Curriculum planners hope that as a result of the new programme, students will be willing and able to think in terms of mathematical models, and perhaps create other models similar to those they have been taught, with the result that students become more creative in adjusting to a shifting world.

Although it is true that the new programme includes the teaching of definitions, postulates, and axioms as part of a mathematical system, these elements are not taught as absolute and final 'truths'. Rather, students are exposed to practical learning experiences which exemplify the characteristics of the mathematical system being taught. In this way, the students gradually come to understand how the definitions, postulates, and axioms were developed. They do not simply learn by rote and memorize formulae to apply to the solution of particular problems. Thus, the new programme stresses the search for, and study of basic principles and the structures of the physical world.

The working procedures of the mathematics team

The Development School Project involves the co-operation of several teams, one of which is responsible for the mathematics programme. This mathematics team has two subgroups. The first is the core group, whose job it is to develop the new programme. The second is a consultative group, composed of people representing the various interests of teachers, university lecturers, and members of society who are interested in the reform of mathematics teaching. The task of the second subgroup is to assess the ideas proposed by the first group, and to propose such revisions as are required.

Each stage of the programme involves a number of steps. First, the core subgroup, made up of three programme developers, devises a set of proposals. These are then discussed with members of the second subgroup, made up of four reviewers and several consultants who are expert in mathematics at the university level. This larger group assesses the set of proposals, and recommends improvements which the core group will seek to incorporate into the proposals. The revised set of proposals is then handed over to writers, some of whom are members of the core group and some who are not. The writers' job is to convert the set of proposals into a set of learning modules, ready for use in the classroom.

The completed modules are then tried out in the classrooms of the eight experimental Development Schools, and results are reported to the mathematics planning team. The reports provide information on how well the time allotments assigned to the modules fit the working styles of the students, on how well the students have performed on the test at the end of the module, and on what difficulties the students encountered in studying the modules. The module writers then use this feedback information to revise the modules.

To the extent that the modules have engaged students in active learning, they appear to have been quite successful. Students have attentively performed the tasks suggested in the reading materials. They have worked together diligently on co-operative assignments, and they willingly assist each other with difficult portions of an assignment.

Mathematical concepts — decisions on the inclusion and the exclusion of topics

As has been said, the old programme placed undue emphasis upon computation, and on quick tricks for finding solutions to problems. It paid very little attention to imparting an understanding of the concepts and structures which computation uses. Thus, in formulating criteria for deciding which aspects of the traditional programme should be retained and which should be deleted or revised, the planning team focused on a change of emphasis away from rote memorization and mechanical computation towards an understanding of the concepts and structures. For example, in geometry, applying such criteria has eliminated trick problems involving the idea of congruence. Similarly, most of the traditional problems in trigonometry and in solid geometry concerned with finding points between lines and planes have been deleted.

As an example of additional material designed to provide a broader understanding of mathematical structures, the topic of *sets* is now dealt with before many of the traditional items in mathematics are taught. So, with an understanding of basic set theory, students are better equipped to comprehend the relationships between the traditional topics. Furthermore, the various number systems have been given prominence, and new topics have been introduced, such as matrices, probability theory and statistics, mathematical deduction, linear programming, approximation techniques, and others.

Conflicting opinions

Some people have disagreed strongly with the introduction of these newer approaches to mathematics teaching. Despite their objections, the planning team is convinced that the newer approach is more

efficient than the traditional one. In the past, the relationship between algebra, geometry, trigonometry, and arithmetic were far less apparent than they are in the new programme. The new emphasis on broad concepts which bind the several branches of mathematics into an interrelated whole should help students to perceive the integrated nature of mathematics. Students should consequently come to realize that the processes of thought which mathematics employs can be applied to all branches of the subject. These processes are essentially those of logical thinking, of decision-making, of systematic study and of analysis. Students should also learn that explicitly stated hypotheses are needed for the clear analysis of problems, and that a mathematical system consists of definitions and hypotheses upon which one builds a set of consequences by means of logical reasoning. Rather than teaching algebra, geometry, trigonometry, and arithmetic as separate subjects, the new programme arranges concepts in a sequence in which one topic forms a logical foundation for the next.

Evaluation and Achievement

The process of validation

The mathematics modular programmes are now being tried out in eight pilot schools along with four other subject areas, social studies, Indonesian language, English language, and Pancasila moral education. The process has been going on since 1975. The results of the evaluation will constitute a basis for the improvement of the materials and their implementation in the classroom.

Both objective and subjective data are collected for each module. The objective data consist of: the amount of time spent by students in completing the module; the mean score obtained by students on the test given at the end of the module; and the percentage of correct responses to every item of the after-test. The subjective data consist of: the difficulties encountered by teachers in using the module; the various types of teaching/learning processes employed in the lesson; and teachers' opinions on the adequacy of the printed module. By analysing the report on each module, a writer is able to identify those parts of the module which need both major and minor revisions.

The first modules to be tried out were those for grades 1, 4, 6 and 9. These have now been revised for a second try-out in 1979. The second group of modules, for grades 2, 5, 7 and 10, were tried out in 1976. These are being revised in 1979 for a second try-out in 1980. The third group, for grades 3, 8 and 11, will pass through the same process one year later.

The revision of modules not only involves improving the modules themselves, but also the development of guidelines for teachers, particularly on how to present the module in the classroom.

Evaluation Procedures

Evaluation has three main purposes: to acquire the information needed to improve the modular materials; to acquire the information needed to improve the operation of the programme and the training of teachers; and to acquire information on how far the whole programme has achieved the objectives for which it is designed.

In light of these purposes, the scope of evaluation covers both the outcomes, and the instructional process of the system being tried out. The outcome of the system is judged in terms of how well the objectives of each module, as well as those of the overall programme, are being realized. The process of instruction is judged in terms of its consistency with the plan, and its manageability by teachers and students.

To acquire information, two major procedures have been used: test and non-test procedures. Tests are used to evaluate student achievement at the end of each module, at the end of each semester, at the end of each year, and on completing the whole programme. Non-test procedures are used to gather information on the process of instruction and the outcomes of the system. These procedures include the use of questionnaires and classroom observation.

The evaluation of the mathematics modular programme involves both a formative and a summative stage.

Formative evaluation has been used to collect the information needed to improve the modules. This strategy has been applied since the beginning of 1975, when trials began. It is carried out mostly by project personnel.

Summative, or overall evaluation is used to collect information regarding the overall effectiveness and efficiency of the whole programme after it has been sufficiently tested and improved on the basis of empirical evidence. This type of evaluation will be conducted in 1981/1982, after which a decision will be made on whether to accept the new system as it is, or to accept it with modifications. The job of evaluating the overall system will be planned and carried out by both internal and external teams of evaluators.

The evaluation conducted at the end of each semester, each year, and at the end of the programme is both summative and formative. It is regarded as summative because it has been used for grading the students. At the same time, however, the results of these evaluations

have been used as feedback to module writers. So, this type of evaluation also plays an important role during the formative stage of the modular system.

Evaluation Findings

The evaluation of the 117 and 172 mathematics modules tried out in 1975 and 1976 respectively indicated the need for further revision. This emerged both from a low level of student achievement and from adverse comments made by the teachers on the printed modules. Nearly one half of the modules need major revisions; the rest need only minor improvements.

Evaluation at the end of the semester

The results of the achievement test conducted at the end of each semester indicate that student achievement in mathematics is still below the expected level of mastery. In the junior secondary school, for example, the average mastery level attained by grade 8 students in 1977 ranged from 26 per cent (pilot school Padang) to 51 per cent (pilot school Semarang), whereas the hoped for mastery level is 75 per cent. Compared to other subject areas, these results are relatively low. However, in the senior secondary school, the average mastery level attained in 1977 by grade 11 students (science and mathematics group) was relatively higher than in the junior secondary school. This ranged from 43 per cent (pilot school Bandung) to 72 per cent (pilot school Padang). Compared to other subject areas, these results are not quite so low.

The evaluation of the instructional process

The application of the modular system has influenced the behaviour of both teachers and students.

The results of classroom observation suggest that more than 30 per cent of the time allocated for each module has been used by the teacher to work with students individually as well as in small groups. This is a higher percentage of time given to individual needs than is usual in traditional Indonesian classrooms. Most of the observers found out that the majority of students – about three-quarters of them – devoted between 75 and 100 per cent of the time to actively working with the module, either reading the text or working through assignments. In effect, students were engaged in active learning most of the time.

Other observations

The modular instruction project is still in the process of development

in the pilot schools, where the first try-out of the modules has been taking place since 1975. The revised modules will be tried out for the second time in the eight pilot schools during the period 1979 to 1981. Since the beginning of 1977, however, the use of modular materials has been extended into the system at large, and schools, other than pilot schools, have been trying out a limited sample of those modules which appear to be working well. The main purpose of this extension is to find out how well the new system can operate within the ordinary school setting.

It is believed that more effective training for mathematics teachers is still needed in order to make the project more successful in the future.

Secondary School Mathematics in Japan

The purpose of this study is to explain the current features of secondary school mathematics in Japan. But it is necessary, first, to create the context, namely the whole system of schooling. This will now be described. Where figures are quoted, the source is the Statistical Handbook of the Ministry of Education, 1978.

The system of schooling in Japan

Schooling is compulsory from the age of 6. It begins in a six-year elementary school. On completing the elementary course, all children move on to three years of lower secondary education. This sequence of nine years of schooling is compulsory for all, and the attendance rate is 99.9 per cent. Of those who complete the lower secondary course, the greater proportion go on to upper secondary schooling, where, in 1977, the attendance rate had reached 93.1 per cent.

In Japan, there are three types of higher institutions: university, junior college and technical college. Technical colleges require for admission the completion of lower secondary schooling, and offer a five-year training programme for technicians. At present, there are about 10,000 students in each grade. This is only 1 per cent of the graduates of the lower secondary schools. Graduates of the upper secondary schools may go on to the universities and the junior colleges.

Schools run by the central government, local government, or non-governmental co-operatives are respectively called national, public or private schools. The number of schools and of students and their relative proportions in 1977 are shown in Table 1.

As Table 1 shows, there are relatively few national or private elementary and lower secondary schools. Indeed, the number of pupils in these elementary schools accounts for 1 per cent or less of the total of the elementary school population, while in lower secondary schools this rises to a little over 3 per cent. But, at the upper secondary school

Table 1.

		Total	National (per- centage)	Public (per- centage)	Private (per- centage)
Elementary schools	Schools	24,777	0.3	99.1	0.7
	Students	10,819,656	0.4	99.0	0.5
Lower secondary schools	Schools	10,723	0.7	94.2	5.1
	Students	4,977,108	0.7	96.1	3.1
Upper secondary schools	Schools	5,028	0.3	75.3	24.4
	Students	4,381,062	0.2	71.0	28.8
Universities	Schools	431	20.4	7.7	71.9
	Students	1,839,363	20.8	2.8	76.4
Junior colleges	Schools	515	6.2	9.3	84.5
	Students	374,244	3.7	5.0	91.3

level, private schools account for 24.4 per cent of the total, while the number of pupils they cater for is 28.8 per cent of the total. However, at the university and junior college level, the part played by private schools contrasts markedly with their earlier role. The consequences are, as is shown in a later section, severe competition in the examinations for entrance into institutions of higher education, notwithstanding the fact that the institutions are sufficient in number to absorb the students. Furthermore, these circumstances have an effect on the organization of courses of study in the secondary schools.

The Existing Curricula for the Lower and Upper Secondary Schools

The main features of education in the lower and upper secondary schools may be summarized as follows:

The General Framework

The subjects to be offered, and the standard number of teaching hours per year for each subject in elementary and lower secondary schools, are laid down in the Ministry of Education Ordinance. For upper

secondary schools, the same ordinance lists only the subjects.

The basic framework, in accordance with which each school organizes its own curriculum including the specification of the objectives and the standard content of the teaching in each subject, is outlined in the national *Course of Study*, a guide compiled by the Ministry of Education for each of the three school levels. The methods to be used in teaching the content are mostly left to the judgement of the teachers except for a few principal considerations which are mentioned in the *Course of Study*. Hitherto, the *Course of Study* has been revised at approximately ten year intervals.

When the *Course of Study* is due to be revised, the draft plans for a revision are made co-operatively by specialists in the Ministry of Education and a committee composed of scholars, intelligent layment, school teachers, school principals and educational administrators, appointed by the Minister of Education, and announced officially by him for public criticism. Final decisions for a new *Course of Study* are made by the Minister, who takes into account the various suggestions set out in the draft plans.

Textbooks

Most of the textbooks used in schools are compiled by commercial publishers in accordance with the requirements of the *Course of Study*. They are then published for school use, providing they have the requisite authorization of the Ministry of Education. Each school adopts only one textbook for each subject. The choice from among the several available textbooks is left, in the case of the public elementary and lower secondary schools, to the municipal boards of education and, in other cases, to each school.

An overall view of the present *Course of Study* of mathematics in lower secondary schools

The present *Course of Study* was made public in 1969, and made effective as from 1972. In 1981, it will be superceded by the next *Course of Study*, revised in 1977.

The standard number of school hours (a school hour is 50 minutes) allotted to mathematics is four per week in each grade. The school year comprises thirty-five weeks or more.

The subject name for mathematics education in the lower secondary school is *Sūgaku* (Mathematics). The subject is not divided into separate branches such as algebra or geometry. The teaching content is classified, grade by grade, into five areas as follows:

- A. number and algebraic expression;
- B. functions;
- C. geometric figures;
- D. probability and statistics;
- E. sets and logic.

By the end of the ninth grade, all pupils in lower secondary school are expected to have covered the following content of mathematics:

- (a) In area A, algebra, up to the solution of quadratic equations, within the domain of rational numbers and their associated square roots;
- (b) In area B, an introduction to the concept of a function up to the simple quadratic function;
- (c) In area C, intuitive concepts of space figures, properties of plane figures up to the theorem of Pythagoras, the angle theorems of circles by deductive methods, and some topological properties through an intuitive approach;
- (d) In area D, descriptive statistics, including the concepts of mean, standard deviation, and correlation exhibited diagrammatically, together with concepts of probability;
- (e) In area E, concepts of sets and logic, including the terms and the symbols associated with them, as are needed to facilitate the learning of the content of the foregoing areas A, B, C, and D.

The Broad Objectives of Lower Secondary School Mathematics

The general aim is to foster in pupils the ability to think mathematically and logically, to help them to grasp various phenomena, and to enable them to consider and to handle them in a unified and developmental way. To this end, the particular aims should be to help pupils:

- 1. to deepen their understanding of the basic concepts, principles, and rules relating to quantities, to geometric shapes, etc., so fostering their ability to develop more advanced ways of treating and disposing of these things in a mathematical manner;
- 2. to acquire the basic knowledge of, and to master the basic skills relating to quantities and geometric shapes, so developing the ability to make accurate and efficient use of these skills and understandings;
- 3. to help pupils towards a deeper understanding of the significance of using mathematical terms and symbols, so fostering in them the ability to use the terms and symbols simply and precisely when

- considering the nature of and the mutual relations between quantities, geometric figures, etc.;
4. to develop the ability of so seeing through a problem that they can consider it logically, and deal with it, presenting the solution in a manner which is consistent with the way in which the problem was presented.

The content and the objectives stated above are the same for all pupils, irrespective of the individual differences in their aptitude or their future career intentions. All pupils, therefore are expected to follow the same course throughout their three-year course of mathematics. In the *Course of Study*, it is suggested that individual differences should be catered for by differentiating the depth of treatment of the content. The range of topics listed above is, however, the same for all pupils.

An overall view of the present *Course of Study* of mathematics in upper secondary schools

The present *Course of Study* was made public in 1970 and became effective as from 1973. It is, however, scheduled to be replaced in 1982 by a new *Course of Study*, revised in 1978.

The subject name for mathematics education in the upper secondary school is also *Sūgaki* (Mathematics). It is not, however, organized on a grade system, as it is in the elementary and lower secondary schools. Instead, the subject is organized according to a sequence of short courses, some being compulsory, others being optional.

The names of the various short courses that are on offer, and the standard number of school hours allotted per week to them, are as follows:

General Mathematics	6
Mathematics I	6
Mathematics IIA	4
Mathematics IIB	5
Mathematics III	5
Applied Mathematics	6

In addition, there is a specialized course, the so-called "Science-Mathematics" course, provided for pupils whose curriculum is specially biased towards natural sciences and mathematics. The course is made up of special short courses other than those listed. One of the ingredients, for example, is "Integrated Mathematics". This is studied for thirteen to eighteen school hours per week in the eleventh and twelfth grades, after taking Mathematics I in the tenth grade, and a

course in “Computer Mathematics”, which takes up two school hours per week for a year.

The rubrics which apply to the short courses listed in the penultimate paragraph are as follows:

1. All pupils are required to take either General Mathematics or Mathematics I; the others are optional;
2. Mathematics IIA, IIB, and Applied Mathematics should be studied after Mathematics I; and Mathematics III should follow Mathematics IIB.
3. The course comprising Mathematics I, IIB and III may be regarded as an academic one;
4. General Mathematics and Mathematics IIA may be regarded as a non-academic course, as the emphasis in these courses is on intuitive approaches and practical activities such as experiments or actual measurements;
5. Applied Mathematics is intended mainly for pupils in vocational courses, and for pupils who take a practical course; they learn advanced topics needed to support their specialized subjects.

In the academic course, comprising the sequence Mathematics I, Mathematics IIB and Mathematics III, topics from algebra, geometry, analysis, probability and statistics are integrated into a coherent system, with material in set theory and logic being appropriately related to them whenever possible. The detailed aims follow.

Algebra in Mathematics I

1. To understand the real numbers and the operations associated with them.
2. To realize that polynomials and rational expressions are similar in structure to integers and rational numbers respectively, and to learn to compute with simple polynomials and rational expressions.
3. To develop an understanding and an ability to work with quadratic equations and inequalities with real coefficients, and with simple absolute inequalities.
4. To deepen the understanding of numbers by extending the range to include real and complex numbers, and also to appreciate that equations can have their solutions in the extended range of numbers.

Algebra in Mathematics IIB

5. To understand matrices and their operations.

6. To understand that by using matrices a system of simultaneous linear equations can be expressed as one equation, and to understand how linear transformations are related to matrices.
7. To understand the principle of mathematical induction by applying it to the binomial theorem and to sequences of numbers; also, to understand the procedure and the significance of defining things inductively by stating the characteristics of a simple sequence of numbers.

Geometry in Mathematics I

1. To understand the concept of a vector in a plane, and to master the operations of addition and subtraction of vectors, and of multiplying a vector by a real number.
2. To learn how to describe a plane figure in relation to a co-ordinate system by means of an equation.
3. To learn how to study the properties of plane figures, and the relationships between them by referring to their equations.

Geometry in Mathematics IIB

4. To understand the role of axioms in mathematics and to understand the meaning of axiomatic construction by a study of plane geometry.
5. To extend the notions of co-ordinate and vector systems to three dimensions, and to understand that vectors are based on the same concept both in the plane and in space.

Analysis in Mathematics I

1. To understand the meaning of mapping, and the composition of mappings and inverse mapping.
2. To perceive functions as mappings.
3. To understand the characteristics of simple functions, such as the quadratic function, simple rational functions, and exponential and logarithmic functions.
4. To understand the characteristics of trigonometric functions and their periodicity.

Analysis in Mathematics IIB

5. To understand the concepts of the differential coefficient and the derivative, and to be able to compute and to apply derivations of simple integral functions.

6. To understand the concept of integration, and to integrate simple integral functions and apply them to problems.

Analysis in Mathematics III

7. To understand the idea of a limit, as it applies to an infinite sequence of numbers, and thereby to understand the meaning of an infinite geometric series.
8. To be able to calculate the derivatives of simple elementary functions and apply them to problems, as follows:
 - (a) Differentiation of the quotient of two functions, such as:
 $y = (ax + b) / (x + c)$ and $y = (ax + b) / (x^2 + 1)$;
 - (b) Differentiation of composite and inverse functions, such as:
 $y = x^n$ (n is a rational number),
 $y = \sqrt{ax + b}$ and $y = k \sqrt{a^2 - x^2}$;
 - (c) Derivatives of trigonometrical functions;
 - (d) Derivatives of exponential and logarithmic functions, such as:
 $y = e^{ax}$ and $y = \log(ax + b)$;
 - (e) The second derivative.
9. To deepen the understanding of integration, and to calculate the integrals of simple elementary functions and apply them to problems, as follows:
 - (a) Integration by parts, but limited to only one such application;
 - (b) Integration by substitution, but limited to such types as:
 $t = (ax + b)$ or $x = a \sin \theta$.
10. To appreciate the significance of differential equations, and to solve such equations as $dy/dx = ky$ (k is a constant).

Probability and statistics in Mathematics I

1. To understand the meaning of probability and its basic laws.
2. To become proficient in determining the probability of the occurrence of events in simple situations.

Probability and statistics in Mathematics III

1. To gain a clear idea of the meaning of a population and a sample, and of the meaning of a probability distribution.
2. To be familiar with the binomial and the normal distributions, and to use them in simple cases.

3. To understand the basic ideas of statistical inference, such as estimation and test.

Sets and logic in Mathematics I

1. To deepen the understanding of sets, propositions, compositions and relations among propositions.
2. To relate the materials in sets and logic to the materials in other parts of the courses as background for facilitating the learning of their topics.

The Broad Objectives of Upper Secondary School Mathematics

The main aim is to help pupils to develop their ability to formulate in mathematical terms problems which occur in a variety of phenomena, and to solve them in logical, systematic and integrated ways. The course also aims at helping pupils to develop an appreciation of the importance of the role of mathematics in society. To this end, the specific aims should be to help students:

1. to gain an understanding of the fundamental concepts, laws, principles, and relationships in mathematics, so as to develop the ability to think about and deal with things in more advanced mathematical ways;
2. to acquire basic knowledge and to master basic skills combined with an ability to apply them efficiently and accurately;
3. to deepen their appreciation of the significance of using the terms and symbols of mathematics, and to develop their ability to express mathematical properties and relations concisely and clearly, thereby to promote a readiness to use them in real situations;
4. to develop insight and the ability to think logically through abstract ideas, and, in the case of practical problems, to cultivate a readiness to check results for their reasonableness;
5. to understand the way in which a mathematical approach may be used in the systematic organization of procedure.

By scrupulously examining the above objectives of upper secondary school mathematics, they would be found to be a natural and coherent development of the objectives of lower secondary school mathematics already described.

The social background to the recent urge to reform the existing national curricula in Japan

As has already been pointed out, revisions have recently been made to the existing national curricula. These will come into effect as follows:

The Elementary School (Grade 1 – 6) revised in 1977, to operate in 1980

The Lower Secondary School (Grade 7 – 9) revised in 1977, to operate in 1981

The Upper Secondary School (Grade 10 – 12) revised in 1978, to operate in 1982

Behind these changes are a number of social problems which deserve description.

The higher rate of school attendance

A rising rate of school attendance in compulsory education has been a national tradition since the beginning of the twentieth century. It has continued to the present, even though compulsory education was extended to lower secondary school by the post-war innovations of 1947. Almost 100 per cent of the age groups of compulsory education are now attending school.

Likewise, there has been a rising percentage in the numbers of those who go on to upper secondary schools after completing lower secondary school, beginning with 42.5 per cent in 1950, increasing to 57.7 per cent in 1960, 82.1 per cent in 1970, and 93.1 per cent in 1977.

This trend has been followed by a rising rate of entry into universities or junior colleges, as follows: from 10.1 per cent in 1955 to 10.3 per cent in 1960, 24.0 per cent in 1970, 35.3 per cent in 1974, and 38.3 per cent in 1977.

These trends appear to reflect some correlation between popular belief in the importance of school education and popular awareness of a growth in economic development.

Inevitably, however, such sharply increasing attendance rates have introduced many low achievers into the higher levels of education, and the question of how to educate them suitably has become a severe social problem.

Antipathy to differentiation and bias to academic courses

There has also grown up a popular antipathy to differentiated curricula, and, with it, a demand for equal opportunity to enter higher education courses, notwithstanding the wide difference in aptitudes among

students. For example, in the curriculum revised in 1962, the mathematics laid down for the ninth grade was divided into 'required content' and 'selective content'. The intention was to make it possible for some pupils to limit their learning strictly to what was 'required'. But, in practice, very few schools were prepared to provide a course which covered the 'required' content only. Consequently, this provision was discarded when the time came for the next revision in 1969.

One more example may be cited. In upper secondary schools, the proportion of pupils who select each of the short courses in mathematics may be estimated by the demand for copies of textbooks. By this criterion, the options work out as shown in Table 2.

Table 2

Grade	Option	Percentage of Students selection Option
In the tenth Grade	General Mathematics	2
	Mathematics I	98
In the eleventh Grade	Mathematics IIA	25
	Mathematics IIB	50
	Applied Mathematics	10
In the twelfth Grade	Mathematics III	30

The above estimation indicates that the percentage of pupils who select non-academic short courses, such as General Mathematics and Mathematics IIA, is much lower than that of pupils who select academic short courses.

All of the above point to the prevalent tendency of parents and pupils wishing to follow an academic course in mathematics. Many people seem to think that it is good for their children to receive a university education if possible. Accordingly, courses which lead elsewhere are judged undesirable.

But, such extreme parental bias towards academic courses has laid upon pupils burdens which for some are too heavy to bear, especially the slow learners. How to deal with this state of affairs has become another severe social problem to be settled.

Heavy competition in the entrance examinations

Selection is by examination for entrance into national or private lower secondary schools from public elementary schools, into upper secondary schools from lower secondary schools, and into universities

or junior colleges from upper secondary schools. Only for entrance into public lower secondary school from elementary school is there no selection procedure.

These entrance examinations are of a competitive nature in that a limited number of places are available and the competition for entrance into schools or universities with a high social prestige is accordingly very severe. Though the percentage of successful candidates quoted earlier might look higher than might be expected from what has just been said, it must be realized that the high figures result from candidates who failed to achieve their first choice being obliged to take offers from the second or third best.

As mathematics is usually one of the key subjects in these examinations, schools tend to prepare pupils for selection examinations in ways which militate against their providing a normal and desirable form of mathematics teaching.

With the public upper secondary schools, the examinations for entry are administered by each prefectural board of education, and are open to candidates from all the schools under its control. In other cases, each school, university, or junior college administers its own examinations separately, and sets problems independently. Commonly, when a new type of problem is set in some examination, it is likely to become a fashion in other examinations in subsequent years. This tendency, in turn, has its effect upon lower schools. It augments the work done by way of preparation for examinations at the expense of the normal course of study. It also places such an importance upon the coverage of different types of problem solving as to result in a neglect of other important aspects of mathematics education.

Those who fail the examination for entry into their desired university usually continue, after graduation, to prepare to try again the following year. These students are called *ronin*, which means a masterless *samurai*, or feudal warrior in Japan. In many large cities, there are private schools, outside the regular system, who recruit the *ronin* and prepare them for the next examination. These schools are called *Yobiko* (preparatory school). Many of them are prospering.

In addition, tutorial classes, called *Gakushu Juku*, also private enterprises, are becoming popular in cities. In these classes, pupils of elementary or lower secondary schools can receive special tutoring, by attending outside their regular school hours, to avoid their becoming future *ronin*.

In order to mitigate the hardship presently prevailing in the competition for entry to the universities, a new national institution, called the University Entrance Examination Center, was established in May,

1977, and a new system of entrance examinations, based upon its services, applied to all national and public universities throughout the country as from January 1979. Under this system, the Center will become responsible for the evaluation of the scholastic attainment of all candidates for admission to university in the general and basic learning content of upper secondary school education. The final decision to admit students will remain with each university. Some will decide after considering only the achievements of candidates as assessed by the Center; others will decide after adding to the Center's assessment the results obtained in a second examination administered by the university itself.

Efforts made to assist teachers to train themselves for the new existing course of study of mathematics, and social criticisms of the new curriculum

In revising the *Course of Study* of secondary school mathematics, a number of modern mathematical concepts were introduced. These included sets, mapping, probability, logic, structure, matrices and so on. With these innovations, it was expected that, when the aims were fully realized, further progress would have been made in mathematics education in Japan. In order to ensure the success of the curriculum reform, the following opportunities for teachers to train themselves were given:

- (a) workshops were held on a nation-wide scale under the auspices of the Ministry of Education;
- (b) regional workshops were conducted by the Board of Education or the Educational Research Center in each prefecture;
- (c) meetings were called to receive the results of research in schools designated by such authorities as the Ministry of Education or the Board of Education;
- (d) educational television programmes were broadcast by Nippon Hoso Kyokai (NHK — Japan Broadcasting Corporation) to help teachers to understand the new curriculum through the demonstration of model lessons on the modern concepts and ideas followed by discussion among the participants in the programme;
- (e) guide materials were prepared and edited by the Ministry of Education;
- (f) national and regional meetings on mathematics education were arranged by such private study groups as the Japan Society of Mathematical Education (JSME).

As a consequence of these earnest efforts of teachers to improve their methods of teaching and to introduce the modern concepts, good results were expected. Unfortunately, severe criticisms of the new curriculum began to appear soon after it was made public. The background to this reaction may already be inferred from what has been said in a previous section. The various causes of discontent were:

- (a) the new curriculum does not cater for the many low achievers in higher school education arising from the steep increase of school attendance rates;
- (b) the constitution of the short courses in upper secondary school mathematics does not take into account the pupils' antipathy to differentiation or their bias towards following an academic course;
- (c) the content of the new curriculum appeared to make such demands upon the pupils that more than ordinary efforts were expected from them.

Severe criticisms were especially levelled at the course of study of mathematics, both by parents and teachers, and by some mathematicians. The critics insisted that it was not adequate to introduce new mathematics into the curriculum — the ideas were neither basic, nor easy for pupils to learn.

Furthermore, such media of mass communications as the press, television and various weekly magazines frequently announced that the new curriculum was responsible for a diminution in the ability of pupils to perform calculations.

In order to meet this criticism, the Ministry of Education set in motion a revision of the existing curricula of elementary and secondary schools.

General principles governing the revision of the secondary school mathematics curriculum, 1977 and 1978

The guidelines for revision adopted by the Curriculum Committee nominated by the Ministry of Education are as follows:

- (a) to lighten the pupils' load of learning by reducing the number of school hours assigned to all subjects in lower secondary school by about 10 per cent on the average;
- (b) to fit the content of the teaching into the fewer school hours according to the following principles:
 - 1. by avoiding fruitless duplication of the contents of mathe-

mathematics from elementary school to upper secondary school, and by teaching concentrically;

2. by transferring some content to a higher grade or by cancelling some;
- (c) by laying stress upon basic knowledge and the basic skills of mathematics, so as to develop an ability to think mathematically and to tackle problems in a mathematical way.

The main improvements to the *Course of Study* for lower secondary school mathematics

The essential improvements may be summarized as follows:

- (a) the statement of objectives for the subject is reduced to the following concise form in contrast to that in the existing Course of Study: "To deepen the understanding of the basic concepts, the principles and the laws of quantities and of geometric figures, and to develop the ability to express and deal with situations in a mathematical way, and thereby to cultivate the ability to handle them efficiently".
- (b) the former basis of sets and of logic is no longer mentioned in the Course of Study; it is left to teachers or to writers of textbooks to work these in themselves;
- (c) some topics have been deferred, and will be treated in a higher grade as follows:
 1. inequalities: from seventh grade to eighth grade
 2. general definition of a function: from seventh grade to ninth grade
 3. descriptive statistics: from seventh grade to eighth grade
 4. probability: from eighth grade to ninth grade
- (d) some topics have been dropped entirely from the syllabus of the lower secondary school:
 1. structures of a set of numbers;
 2. simultaneous linear inequalities in two variables;
 3. the cubic function, e.g. $y = ax^3$;
 4. inverse functions;
 5. the transformation of geometric figures;
 6. permutations and combinations;
 7. expectation, standard deviation and correlation.

It should, however, be noted that almost all of the above has been

transferred to some part of the Course of Study for upper secondary schools. In the newly revised Course of Study of lower secondary school mathematics, the prescribed number of school hours per week are 3, 4, 4 for the seventh, eighth and ninth grades respectively. There will thus be one hour less per week of mathematics in seventh grade than in the existing Course of Study.

The main improvements to the *Course of Study* for upper secondary school mathematics

The revised curricula, made public in June 1978, were originally in draft, so as to invite public criticism. It is, however, expected, from past experience, that the Minister of Education will pronounce the draft plan with slight modification to be the new *Course of Study*, to become effective from 1982.

The essential improvements to the Course of Study may be summarized as follows:

- (a) The statement of objectives for the subject is reduced to the following concise form, as in the Course of Study of lower secondary school mathematics: "To deepen the understanding of the basic concepts, the principles and the laws, and to develop the ability to consider and deal with the situations in a mathematical way, by systematic organization, thereby to cultivate the ability to handle them efficiently";
- (b) The names of the short courses which compose the main course, and the number of allotted school hours per week are as shown in Table 3.

Table 3. Allocation of hours among Short Courses

Mathematics I	4
Mathematics II	3
Algebra and Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability and Statistics	3

The rubrics which apply to the above courses are specified as follows:

1. Mathematics I is compulsory for all pupils in the tenth grade; the others are optional;
2. any one of Mathematics II, Algebra and Geometry, Basic

Analysis, and Probability and Statistics can, as a rule, be studied after Mathematics I;

3. Differential and Integral Calculus should, as a rule, be studied after Basic Analysis;
4. Mathematics II is intended to be a non-academic course, but pupils who complete it may be allowed to take other optional courses if they wish.

For the specialized 'Science-Mathematics' course, two short courses, named 'Science Mathematics' and 'Integrated Mathematics' have been prepared. The former course covers roughly the same ground as Mathematics I, but it includes some computer mathematics. The latter course comprises an integration of the contents of the optional courses in the ordinary course. In this course, however, pupils are trained, through being given assignments and by working in seminars, to become more self-reliant in their learning habits.

The aims and content of the revised short courses can now be described.

Mathematics I

- (a) Aim: To deepen the understanding of numbers, of algebraic expression and of geometric figures, and to master the basic knowledge and skills associated with them, developing thereby the ability to use them exactly in handling mathematical situations.
- (b) Content:
 1. Numbers and Algebraic Expressions:
 - a) numbers and sets;
 - b) integers, rational numbers, real numbers;
 - c) polynomials, rational expressions;
 - d) quadratic equations, simple equations of higher degree, simultaneous equations;
 - e) quadratic inequalities;
 - f) algebraic expression and proof;
 - g) terms and symbols: discriminant, the imaginary number, i , complex number;
 2. Functions:
 - a) quadratic functions;
 - b) simple rational and irrational functions;
 - c) terms and symbols: inverse function;
 3. Geometric figures:
 - a) trigonometrical ratios: sine, cosine, and tangent;
 - b) the sine theorem; the cosine theorem;

- c) plane figures and their equations: the coordinates of a point; the equations of straight lines; the equations of circles;
- d) terms and symbols: $\sin$, $\cos$, $\tan$.

Mathematics II

- (a) Aim: to help pupils to understand the basic concepts, principles and laws in the broader field of mathematics associated with the content of Mathematics I, and thereby make them more aware of the role played by mathematics in society.
- (b) Content:
 - 1. Probability and Statistics:
 - a) permutations and combinations;
 - b) probability;
 - c) statistics;
 - d) terms and symbols: ${}_nP_r$, ${}_nC_r$, factorial n , $n!$, complementary event, expectation, standard deviation;
 - 2. Vectors:
 - a) vectors and their operations;
 - b) applications of vectors;
 - 3. Differential and Integral Calculus;
 - a) the meaning of the differential coefficient;
 - b) derivatives and their applications;
 - c) the meaning of integration;
 - d) terms and symbols: limit, 'lim', indefinite integral, definite integral;
 - 4. Sequences:
 - a) arithmetic series;
 - b) geometric series;
 - 5. Various Functions:
 - a) exponential function;
 - b) logarithmic function;
 - c) trigonometric functions;
 - d) terms and symbols: power, root, $\log_a x$, generalized angle;
 - 6. Computer and Flow Charts:
 - a) the function of an electronic computer;
 - b) algorithms and flow charts.

Algebra and Geometry

- (a) Aim: To help pupils to understand vectors and matrices, to cultivate their abilities to use them, and furthermore, to deepen

their understanding of conics and of figures in space by developing their ability to use co-ordinates and vectors.

(b) Content:

1. Conics:
 - a) the parabola;
 - b) the ellipse and hyperbola;
2. Vectors in a Plane:
 - a) vectors and their operations;
 - b) the inner product of vectors;
 - c) the applications of vectors: equations of straight lines, circles, etc.
3. Matrices:
 - a) matrices and their operations;
 - b) the inverse matrix;
 - c) linear transformation and mapping;
 - d) terms and symbols: A^{-1}
4. Space Figures:
 - a) points, straight lines and planes in space;
 - b) co-ordinate system in space;
 - c) vectors in space including equations of straight lines, planes and spheres.

Basic Analysis

(a) Aim: To help pupils to understand sequences, the exponential function, the logarithmic function, and the trigonometrical functions, and to inculcate the basic ideas of differentiation and integration, thereby developing their ability to use them in case of simple integral functions.

(b) Content:

1. Sequences:
 - a) simple sequences: arithmetic series; geometric series; etc;
 - b) mathematical induction;
 - c) terms and symbols: ' Σ ';
2. Functions:
 - a) exponential function;
 - b) logarithmic function;
 - c) trigonometrical functions: generalized angle and circular measure; trigonometrical functions, their periodicity, and the addition theorems of trigonometrical functions;
 - d) terms and symbols: power, root, $\log_a x$

3. Variation of the values of a function:
 - a) the meaning of differential coefficient;
 - b) derivatives and their applications: derivatives of the sum and difference of functions, and of a function multiplied by a real number; tangent, increase and decrease of the values of function, velocity, etc.;
 - c) integrations and their applications: indefinite integral, definite integral, area, etc.;
 - d) terms and symbols: limit, 'lim'.

Differential and Integral Calculus

- (a) Aim: To help pupils to understand the concept of a limit and to deepen their understanding of the concepts and the laws of differentiation and integration, thereby developing their ability to use them in the case of simple elementary functions.
- (b) Content:
 1. Limits:
 - a) the limit of a sequence;
 - b) the limit of values of function;
 - c) terms and symbols: convergence, divergence, ∞ ;
 2. Differentiation and its applications:
 - a) derivatives: differentiation of the product and quotient of functions; differentiation of composite and inverse functions; derivatives of trigonometric functions; derivatives of exponential and logarithmic functions;
 - b) applications of derivatives: tangent, increase and decrease of the values of function, velocity, acceleration, etc.;
 - c) terms and symbols: natural logarithm, 'e', the second derivative, a point of inflection;
 3. Integration and its applications:
 - a) integration: the meaning of integration, integration by substitution and by parts in simple cases, the integration of various functions;
 - b) applications of integration: area, volume, length of path, etc.; the meaning of a differential equation, including the solving of such equation as $dy/ds = ky$ where k is a constant.

Probability and Statistics

- (a) Aim: To deepen the pupils' understanding of the basic concepts and the laws of probability, and to help them to understand the concept of a probability distribution, thereby developing their ability to think in statistical terms.

(b) Content:

1. The arrangement of data:
 - a) the distribution of a variate;
 - b) the representation of values and of measure of dispersion;
 - c) terms and symbols: variance, standard deviation, ' Σ ';
2. The number of possibilities:
 - a) permutations and combinations;
 - b) the binomial theorem;
 - c) terms and symbols: ${}_nP_r$, ${}_nC_r$, factorial n , $n!$;
3. Probability:
 - a) probability and its basic laws;
 - b) an independent trial and its probability;
 - c) conditional probability;
 - d) terms and symbols: complementary event, exclusive event, independence, dependence;
4. Probability distributions:
 - a) random variable and its probability distribution;
 - b) binomial distribution, normal distribution;
 - c) terms and symbols: expectation;
5. Statistical inference:
 - a) populations and samples;
 - b) the idea of statistical inference;
 - c) terms and symbols: estimation, test.

As the above shows, the content of the revised Course of Study for upper secondary school mathematics is more concisely expressed than it is in the existing course of study. In addition, it is the intention of the authorities to encourage flexibility both of content and of time, so that either or both may be increased or decreased according to the abilities and the aptitudes of the pupils. As has been said, these courses have yet to secure the approval of the Ministry before they become mandatory in 1982. It remains to be seen, therefore, whether, and to what extent, the proposed Course of Study will find favour with the critics of the current courses, will be found workable by the teachers in the schools, and will be more acceptable to students, parents and employers. Time alone will tell.



Recent Developments in Mathematics Education in the Philippines

Collaborative Activity

The mathematical activities presented here may be described as a co-operative venture between the national mathematical societies and the various educational and government agencies which are concerned to improve the state of mathematics education in the Philippines. They do not deal with specific experiments in curriculum development or in teacher-training. This is because analysis has shown that the problems which beset mathematics education in the Philippines are so basic and so pervasive that only a massive integrated effort can produce the 'take-off' effect necessary to bring about lasting improvements. It is understood, of course, that pilot projects in curriculum and teacher-training are necessary, but they cannot at the moment form the focus of effort. The immediate need is for improvements with wider scope, even if these are necessarily of limited quality. In particular, the report will describe two most recent efforts: a survey of the status of teacher education in mathematics, and the teaching of senior high school and freshman college mathematics; and the first Southeast Asian Conference on Mathematical Education held in Manila, 29 May to 3 June 1978, together with the subsequent echo conference held in Iligan City in the southern island of Mindanao, 4 to 9 June 1978.¹

Origins of the Collaboration

The Mathematical Society of the Philippines (MSP) was established in 1972 by a group of concerned mathematicians in order to provide an organization and a focus for the efforts to improve mathematics in the country both in research and in education. In its first few years, the MSP concentrated on trying to develop a 'critical mass' of mathe-

1 Proceedings of the First Southeast Asian Conference on Mathematical Education, Manila, 29 May - 3 June 1978. (To appear in early 1979 and available through the Mathematical Society of the Philippines, c/o Ateneo de Manila University, P.O.Box 154, Manila, Philippines).

maticians, who could teach at a high level, and eventually form a school of research. Eventually, however, the attention of MSP was drawn to the grave problems besetting school mathematics. This shift of attention was brought about by several factors: the experience of many teachers of the deteriorating mathematical abilities of students entering high school and college; the poor background of high school and college teachers of mathematics who were seeking higher degrees; and a study by the Ministry of Education, called SOUTELE (Survey of Outcomes of Elementary Education)².

Among the many disturbing findings of SOUTELE, the most striking was that the subject matter which sixth graders across the nation seem to learn least were the skills traditionally associated with the three R's: 'reading, 'riting and 'rithmetic'. The mean score on achievement tests in all three areas were below 50 per cent, with mathematics the lowest (38 per cent).

In an effort to begin to meet these problems systematically, the Mathematics Teachers Association of the Philippines (MTAP) was revived. Together MSP and the MTAP began the work of seeking serious reforms in mathematics education. The efforts began with nation-wide seminars for mathematics teachers in order to impart needed information and skills, as well as to generate dialogue with them about their key problems and concerns. By late 1976, these efforts led to a conference on mathematical education, held in Manila in late May 1978, and to a decision to make surveys of the status of teachers, of teacher education and of the teaching of senior high school and freshman college mathematics.

Seminars for Teachers

The preparation for the surveys and for the mathematics conference began with seminars held all over the country. In April 1977, a convention on mathematics education at the tertiary level was held at the University of the East. This was followed by six one-day seminar/workshops for teachers of undergraduate mathematics held in leading universities and colleges of Metro Manila. In September, 1977, a workshop for secondary mathematics teachers, supervisors and teacher-trainers was jointly sponsored by the Arellano University, the Ministry of Education and Culture (MEC) Region IV and the Mathematical Society of the Philippines. In preparation for this workshop, a survey of secondary mathematics teachers of the Division of Manila was

² Report on the SURVEY OF OUTCOMES OF ELEMENTARY EDUCATION, SOUTELE, Department of Education and Culture, Manila, Philippines. (Available at the Ministry of Education and Culture, Manila, Philippines).

conducted. In February, 1978, a seminar/workshop on undergraduate and graduate programmes in mathematics was held at the Far Eastern University under the auspices of MSP. Seminars for tertiary teachers were also conducted in Cagayan de Oro City, Marawi City and Baguio City.

Sponsored by MPAT and the MEC Region IV, an *ad hoc* committee was set up to plan activities in secondary mathematics for the region, and to look into the in-service training of secondary mathematics teachers. In the course of several workshops, a small group of mathematics teachers from private and public high schools prepared syllabuses in Elementary and Intermediate Algebra, Geometry and Algebra-Trigonometry. It is proposed to implement those syllabuses in all high schools by the 1980/81 school year.

At the elementary level, teachers from selected schools were invited to several workshops to look into the elementary mathematics curriculum. Two weeks before the First Southeast Asian Conference, Professor Zbigniew Semadeni, Deputy Director of Mathematics of the Polish Academy of Sciences, conducted a seminar/workshop on the teaching of elementary mathematics. Twenty-five selected representatives from teacher-training colleges throughout the country took part in this gathering.

The seminars and workshops took up topics that were particularly important for teachers. They also served as an occasion for the discussion of problems which are central to mathematics education in the Phillipines. The problems discussed were of two kinds: those which beset education in general, such as the low salaries of teachers, their heavy working loads, overcrowded classrooms, lack of time to teach basic subjects, etc.; and problems which are peculiar to mathematics education, such as the poor implementation of new mathematics in the majority of schools, confusion about the essentials in mathematics, and the poor background of teachers.

While efforts to improve mathematics education must aim mainly at problems of the second kind, it was recognized that these would not entirely disappear unless those of the first kind were attended to.

The First Southeast Asian Conference on Mathematics Education

The first Southeast Asian Conference on Mathematics Education was held in Manila from 29 May to 3 June 1978. It was organized by MSP with the assistance of the National Science Development Board, the International Commission on Mathematics Instruction ICMI, the Southeast Asian Mathematical Society (SEAMS) and MTAP.

More than one thousand participants attended the Conference. The great majority of them were from the Philippines. About one half were teachers from colleges and universities; a quarter came from secondary schools; and another quarter from primary schools.

Twenty participants came from other countries. Among these were five official representatives of SEAMS. They came from Hong Kong, Indonesia, Malaysia, Singapore, and Thailand. Others included four Australians, two of whom were from Papua New Guinea, four from Hong Kong, two from Japan, two Americans, two Frenchmen and Prof. Semadeni, whose workshop for teacher-trainers was mentioned above. Notable among the foreign participants was Prof. Shokichi Iyanaga, President of ICMI.

The first day of the Conference was devoted to work in plenary session. Five papers were presented. Dr. Melecio Magno, Minister of Science, spoke on the importance of mathematics for national development, and on the efforts of the Ministry of Science to upgrade mathematics education. Prof. Shokichi Iyanaga, President of ICMI, outlined the role of mathematical organizations in the improvement of mathematical education, with particular stress on the work of the Mathematical Society of Japan. Fr. Bienvenido F. Nebres, S.J., President of the MSP and of SEAMS, described the results of the 1978 conference on Developing Mathematics in Third-World Countries, held in Khartoum, Sudan, pointing out that the Khartoum conference addressed itself to problems similar to those confronting mathematical education in the Philippines. Prof. Jean Dhombres, Scientific Counselor of the French Embassy in Canada, and Prof. Jean Giraud of the Ecole Normale Supérieure of St. Cloud, discussed the professional life and development of mathematics teachers. And Prof. Zbigniew Semadeni, Deputy Director of the Institute of Mathematics, Polish Academy of Sciences, dealt with the training of elementary mathematics teachers.

The Conference then concentrated for two days on the problems which are peculiar to the three stages of education: elementary, secondary and tertiary. Thus, in relation to the *elementary* stage, there was discussion of: the SOUTELE study mentioned above, and its implications for elementary school mathematics; language and mathematics; and the return to basics. In relation to the *secondary* stage, there was discussion of: the findings of national testing programmes (for high school graduates entering college) related to mathematics and mathematics teaching; detailed reports on the development of the secondary mathematics curriculum in Southeast Asian countries, notably Hong Kong; and a report on a study made of the training of secondary mathematics teachers and recommendations on reforms in pre-service training. And, in relation to *tertiary* education, there was

discussion of: the mathematics curriculum and instruction for students of the natural and social sciences and engineering; proposals for reforms in the BS Mathematics programmes and in graduate mathematics programmes; and new areas of applied mathematics (notably, operations research and computer science) and the impact of calculators and computers on the teaching of mathematics.

Another two days were dedicated to small group discussions of recommendations for reforms in the pre-service and in the in-service training of teachers, in curriculum, and in supervision. Summary reports of these recommendations were given at a plenary session on the last day of the Conference.

The Conference closed with two concluding addresses. Fr. Bienvenido Nebres spoke on mathematics in Southeast Asia, discussing the similarities as well as the differences between the Southeast Asian countries, and the plans and prospects for cooperative ventures in the near future. Dr. Juan Manuel, Minister of Education and Culture, closed the conference with words of encouragement and support for the work of the Conference towards the improvement of mathematics education.

The Survey of the Status of Teacher Education and the Teaching of Senior High School and Freshman College Mathematics

In order to provide hard data for the Conference on mathematics education, a research project, supported by the National Science Development Board, was undertaken by MSP. The Director of the research project was Sr. Iluminada Coronel, F.M.M. The outcome cited below will be limited to the status of teachers and teacher education. The full study was published early in 1979.

The survey was conducted in Metro Manila and in 25 provinces. It involved a total of 126 schools, 130 administrators, 643 mathematics teachers, and 2354 students (485 math majors, 1869 non-math majors). The salient features of the study follow.

The Pre-Service Training of Mathematics Teachers

Sixty-five per cent of the teachers have only an undergraduate degree. Of these, less than one half obtained their degrees in mathematics (AB Math, BS Math, BSE Major in Math). The rest are general AB or BS or BS (Engineering) degree holders. Aside from the general weakness in mathematics subjects of the BSE programme, a recent survey showed another disturbing finding: students enrolled in BSE programmes belong mostly to the first quartile, yet only 2.6 per cent of their teachers have an MA or MS in Mathematics; 13 per cent hold an MAT

or MST and 1 per cent a PhD in Mathematics. This weakness in pre-service qualification is underlined by weakness in the content of courses taken in their undergraduate and graduate programmes. Very few teachers have followed courses beyond calculus. Only 39 per cent have taken Linear Algebra, 28 per cent Abstract Algebra, 16 per cent Complex Analysis, 11 per cent Advanced Calculus, etc.

The In-Service Training of Mathematics Teachers

The outlook on in-service training is equally bleak. Some 59 per cent of the teachers interviewed had not attended a mathematics seminar in the last three years; 88 per cent do not subscribe to a professional journal; 70 per cent do not belong to a professional organization. The professional indifference is not surprising in the light of the responses of administrators to questions on providing teachers with incentives to pursue their mathematical studies. Thus, 82 per cent give no reduction of teaching load; 75 per cent give no increase in pay after graduation; 92 per cent give no promotion in rank after graduation.

These problems are recognized by teachers and administrators. For example, 61 per cent of the teachers confess that their fundamental teaching problem is their weak knowledge of mathematics. Administrators likewise recognize the lack of good math teachers: 71 per cent agree that there is an inadequate number of them; 60 per cent say that they need additional mathematics teachers; 75 per cent experience difficulty in recruitment. As a result, heavy overloading occurs.

Confirmation of these findings emerged from the reports of the various working groups in the Conference, and a number of reasons were cited for the lack of qualified mathematics teachers in the Philippines: the poor quality of the programmes in BS Math, BSE majors in mathematics and the various graduate programmes for mathematics teachers; the lack of motivation for continuous professional development on the part of many teachers; the lack of incentives given by schools to teachers who pursue higher studies; and the lack of in-service training programmes.

The Working Conditions of Mathematics Teachers

The report also starkly portrays the very poor working conditions of mathematics teachers.

About 72 per cent teach at least twenty hours per week. Of these 13 per cent have an average weekly teaching load of twenty hours, 21 per cent twenty-three hours, 11 per cent twenty-six hours, 16 per cent thirty hours. Only 10 per cent have a weekly teaching load of less than thirteen hours.

The monthly salaries range from P330 to P1,500. These represent about one-fifth of what one can get in business or in some sectors of government. In state colleges/universities and a few of the top private schools, the salary range is P950 to P3,000 a month — better than in the other schools, but still way below, and non-competitive with salaries outside schools. To augment their low salaries, about 25 per cent of the teachers interviewed take on non-teaching jobs.

This combination of heavy teaching loads and poor pay has led many teachers to give up the profession. Thirty per cent of the respondents wish to leave teaching, in favour, the majority say, of a better paying job.

These data are again confirmed by the participants in the Conference. In a report given before the plenary session on the last day, one member summarized the problem as the inequity between the work load and the demands of the profession on the one hand and the compensation on the other. One of the recommendations was that the full-time load should be twelve hours a week, for which the remuneration should be a regular monthly salary, paid twelve months a year, and of such an amount as will enable the teacher to live decently and grow in the profession.

Future Directions and Plans

What has been achieved by the seminars, the Conference, the survey and all the work done so far? Both the Southeast Asian Conference on Mathematical Education and the survey of the state of mathematics education and the status of mathematics teachers have helped to focus attention on the problems faced by the Philippines. The survey, in particular, presents the problems in very stark terms. These efforts have also brought together teachers, administrators and key government ministries to seek a more lasting solution to these problems.

To find genuine solutions for all the problems will take a long time. Even if massive support is forthcoming, it is necessary to think in terms of ten to twenty years. In the meantime, a generation of students is progressing through school. With these thoughts in mind, the recommendations divide into three kinds: programmes for immediate implementation, medium-term plans, and long-term plans.

Programme for Immediate Implementation

These include recommendations on in-service training and on the production of materials. Their main purpose is to improve, in no matter how limited a fashion, the present quality of instruction. They reflect

concern for the large number of students currently passing through the school system.

Two-to-six-week seminars during summers, to begin in 1979

These seminars will initially explain to elementary and high school teachers the work already done by various committees on such matters as the development of basic skills, priorities and the time allotment for various topics, and the two-track syllabus recommended for high schools.

They will also provide guidelines and supplementary materials, especially for high school mathematics. And they will initiate continuing in-service training for teachers in the various regions.

Production of materials

These will consist of: materials designed to help in teaching the basic skills; supplementary problems for high school mathematics; and teachers; manuals giving guidance especially on the priority of mathematical topics and on time allotment.

Medium-term plans

The key to the improvement of mathematics programmes is the training of more and better teachers. The medium-term plans accordingly look forward to improved pre-service and in-service programmes, with associated awards of scholarships and other incentives. These must be planned and implemented immediately. In particular, it will be necessary over the next five years to implement curricular reforms for the BS Math, the BSE major in math, the MS, the MST (Master of Science in Teaching), and the MSCT (Master of Science in College Teaching), as recommended at the First Southeast Asian Conference on Mathematical Education.

With the assistance of the Mathematics Consortium of the three leading universities in Manila, and under the guidance of MSP and MTAP, a large-scale programme of training mathematics teachers should be planned and implemented. This should include the provision of summer institutes, a one-year or fourteen-month certificate programme, and a degree programme, which should offer the MS for teachers of higher level college mathematics, the MST or the MAT for high school teachers or teachers of basic college mathematics, and the MSCT for teachers of college mathematics. These various training programmes should first be provided under the auspices of the Consortium mentioned above, and continue, within two to five years, at regional centres, in Northern Luzon, Southern Luzon, Visayas and Mindanao. There are already some suitable candidates for these regional centres.

Long-term plans

Because of the finances involved, improvements to the salary and working conditions of teachers can only be undertaken gradually over a period of years. But, unless they are made, the proposed measures outlined above will not succeed or stabilize.

There is an urgent need to identify scientific talent as early as possible and to develop it. Consequently, more high-quality programmes, such as are offered in the Philippine Science High School, are needed at the elementary, high school, and tertiary levels.

A system of supervision and evaluation must be evolved to monitor instruction in mathematics. Participants in the Conference expressed their views very strongly on the need for drastic reform of the present supervisory system. In particular, separate corps of supervisors are needed for the elementary and secondary levels. Separate corps are also required for science and mathematics. And all supervisors should be genuinely competent.

Implementation of Recommendations

In order to implement a programme of the scope and magnitude envisaged above, it is considered necessary to establish a National Committee on Mathematical Instruction (NCMI), whose function would be to take overall responsibility for the programme, and to make adjustments to it in the light of changing needs. The NCMI should be composed of concerned agencies and constituted at the highest government level. The day-to-day work of seeing that the orders of the NCMI are carried out would be entrusted to an Executive Board. The Executive Board should be composed of some NCMI members, and some full-time staff. Initially, the members of the Board and of the NCMI could be housed at the Consortium of the three universities in Manila, mentioned above.

A major task of the NCMI will be to lay down a timetable for implementation, and to seek the financial resources that will be needed to put the programme into effect. Thereafter, it will be necessary to identify, and, where necessary, to train the personnel required to implement the reforms.

It will take a little time to set up the required machinery. Prior, therefore, to establishment of the NCMI, the various bodies mentioned above can implement many of the recommendations. For example the Ministry of Education and Culture could institute curricular reforms, make improvements to current arrangements for supervision,

and provide incentives for in-service training. Likewise the Ministry of Science could provide funds, and create the organization necessary for Summer Institutes, Certificate Programmes, and Graduate Programmes, while MSP and MTAP could take responsibility for staffing the in-service and pre-service programmes, and for the organization of mathematics teachers.

Ways of Improving Mathematics Teaching in Soviet General Secondary Schools

The scientific and technological revolution currently taking place, with its considerable concomitant differentiation between the sciences, brings to the fore the question of how to educate people who can become active contributors to social progress, with a grasp of the fundamentals of science and the ability to apply them so as to achieve the most rational organization of the production of material and spiritual benefits. Take any sector of the economy, and you find that the worker is increasingly called upon not only to possess basic and specialist knowledge, but the ability to work creatively as well, to show practical initiative, and the capacity to learn and to educate himself continually. It is precisely these qualities which determine a person's ability to adapt successfully to the many-sidedness, the innovative tendencies and the pressures of modern industry.

It is recognized that to prepare young people for creative activity, one must begin at school. That is why a steady improvement in the effectiveness, and the quality of education is the prime concern of the Soviet education system. And this applies as much to the teaching of individual subjects as to general upbringing, careers guidance, extra-curricular activities or any other aspect.

Scientific and technological progress has inevitably made new demands on the teaching of mathematics. In every branch of industry, mathematics has come to play a greater and greater role as a result of its widespread application in recent decades in areas of science, technology and the economy that once offered very little scope for the applications of mathematics.

This has also been accompanied by mathematical methods being widely used in many branches of the economy. The specialist training of engineers and of skilled workers for industry and agriculture is now firmly wedded to the use of mathematical methods, of automation and of computer technology. The result is that specialists must more and more be schooled in algorithmic and computation methods, and be capable of applying mathematics to the solution of practical problems.

All this means that schools must now teach mathematics in such a way as to ensure that not only scientists and engineers, but today's workers as well can operate effectively.

The net result has been a substantial change in the content and in the methods of teaching mathematics in Soviet general secondary schools.

The new syllabus

The changeover to the new mathematics syllabus in Soviet schools is complete. It is now clear that Soviet general secondary education has taken a great step towards raising the standard of mathematics teaching by making better use of the subject for the pupils' development and general education, by getting away from formalistic learning and by arousing the pupils' interest in mathematics, so generating a desire to learn.

Throughout the period of changeover to the new mathematics syllabus, the Russian Socialist Federal Soviet Republic (RSFSR) Ministry of Education Schools Research Institute, other research establishments in the country, institutes of education, universities and institutes of further training for teachers were carrying out a detailed study of how the main notions in the curriculum were actually being implemented, and how work based on the new textbooks was proceeding. They were also taking into account practical suggestions for improving the textbooks and making more effective instruments of learning.

Studies have shown that the changeover to the new mathematics syllabus has been carried out successfully, although there were difficulties to be overcome in the process.

Some difficulties

These difficulties, as well as a certain lack of knowledge, know-how and experience in the children, resulted mainly from inadequacies of the existing mathematics textbooks and syllabuses used in schools.

If we summarize the findings of the studies, the shortcomings of the mathematics syllabuses and textbooks for the sixth to tenth grades can be said to be as follows:

- (1) Syllabuses and textbooks contain too much material. The result is that not all of the pupils are able to acquire a full and firm grasp of the mathematics course in the lesson time available. The teacher likewise is unable to give enough attention to sorting out pupils' problems, or to recapitulating and consolidating the ground already covered.

The syllabuses and textbooks are overloaded because the mathematics course includes a number of items of secondary importance and concepts that play no significant part in subsequent stages. There is also an excessive use of the spiral method of teaching, and, in certain cases, incomplete structuring of the subject matter.

- (2) Wrong methods are used in some sections of the mathematics textbooks, since some of the material — especially that which concerns geometry, algebra and analytical principles — is set out in language that totally fails to take into consideration the pupil's level of maturity. For example, the explanation of some parts of the course is too abstract and, either too lengthy or, conversely, too terse. Some sections in the textbooks contain far too many new concepts. In many cases definitions and theorems are clumsily expressed, and certain topics are set out in such a way that there is no proper balance between essential and non-essential matter.
- (3) Poor use is made of problems and exercises, as can be seen from the fact that in many cases there appears to be no clear principle governing the choice of the problems and exercises for specific topics. The selection of problems is sometimes inadequate for a clear and lasting understanding of the theory. Some of the problems are either trivial or unnecessarily complicated. Sometimes the principle of grading the complexity and difficulty of problems is ignored, and some of the problems are designed to teach skills that have long been out of date.
- (4) The syllabuses and textbooks do not always maintain the necessary continuity between the course in algebra and analytical principles and the course in algebra in the eight-year school, or between the mathematics course for the fourth and fifth grades and the geometry course for the sixth to eighth grades. A certain inconsistency can also be found between the algebra and geometry courses. Moreover, the interrelationships between the branches are not brought out sufficiently and the multi-disciplinary slant of the subject matter needs to be consolidated.

Changes in the System

With the changeover from the Soviet general-school system to universal secondary education, our scientists and teachers are faced with the task of making further improvements to education. Educationists concerned with mathematical education in particular face the heavy and laborious task of improving its quality still further. This will be done in accordance with the main aim of the resolution passed by the Central Committee

of the Communist Party of the Soviet Union and the Council of Ministers of the U.S.S.R. which said that under universal secondary education '... before leaving secondary school, pupils must acquire a sound knowledge of basic science and operational skills for work in the national economy, going as far as learning a particular trade'.¹

We consider that at the present time it would be wrong to make any substantial change in the content of mathematics teaching or the textbooks in use. Syllabuses and textbooks must remain unchanged over the next five to eight years if we are to improve teaching methods. We believe it is essential to establish conditions in which teachers can work productively and creatively by, for example, providing help with methods at the right time, organizing in-service training and producing the necessary methodological, instructional and popular-science literature.

Syllabus adjustment

The drive to improve the present syllabuses and textbooks is taking place in the context of the introduction of universal, compulsory secondary education into the Soviet educational system. The reshaping of the current syllabuses and textbooks must therefore work towards a reduction in the volume of course material. This will enable all pupils to gain a fuller and more lasting understanding of the basics of mathematics. It will also ensure that teaching time is available for improving the practical aspects (and the more general aspects) of the lessons through the setting of questions on the syllabus and through the solving of problems.

In line with the directive issued by the RSFSR Ministry of Education, the schools research institute adopted this approach in establishing norms for the pupils' workload, and in reducing the current mathematics syllabus for the sixth to tenth grades. This syllabus is now covered in 124 hours of class time.

It must be emphasized that the time so gained is wholly allotted (and has already been broadly allocated) to the deeper study of important sections of the course, to revision and the consolidation of the ground covered and to more independent problem-solving work by the pupils themselves.

In picking out syllabus items that could be dispensed with, the criterion was not only practical experience, but a definite set of other considerations. It was decided, for example, to remove from the syllabus a number of topics (such as combinatory analysis) which, although highly significant for a complete mathematical education,

1 *Narodnoe Obrazovanie* [National Education], 1978, No. 3, p. 3.

tend to be rushed in the school course. Other topics (e.g., certain aspects of identity transformations and the method of mathematical induction) were recognized as being too specialized for a mathematics course in general education. The study of certain topics leads to an excessive use of the spiral method (e.g., certain aspects of three-dimensional geometry in the eighth grade.

At the top of the list of essential syllabus items we placed those that constitute both a subject of study and a means of study (e.g., the general and concrete aspects of the concept of a function), together with those that, while representing only a means of study, constitute an effective tool for acquiring knowledge (e.g., graphs of functions, the solving of equations and inequalities, and so on).

Throughout, every effort has been made to avoid lowering the standard of mathematics teaching set by the existing syllabus, and to preserve the structure, conceptual direction and basic content of the mathematics course while retaining within it a broad range of topics which can contribute to the pupils' development and general education.

Comparative tables of the present and proposed syllabuses are annexed (p.83–86). The proposed syllabus does not destroy the integrity and customary continuity of the structure of the mathematics course, but keeps the main pattern of its development. It can thus be introduced in schools without delay, using the existing textbooks (subsequent editions of which will, of course, have to be amended to reflect the abridged syllabus) and without any need to retrain the teachers. The new version of the existing syllabus thus preserves the constancy of the mathematics course as a whole and consequently aims to maintain the relevance of the textbooks.

Contrasting opinions

Regarding the further development of secondary mathematics teaching, two extreme views are held by our educational theorists.

The one extreme is, figuratively speaking, 'a return to Kiselev' — that is a return to the study of the basic principles of mathematics in the traditional sense, recognizing that the set-theory approach to the formulation of the basic concepts in the course is complex and little suited to general education.

The other extreme advocates an even more extensive updating of the course content, permeating it with elements of mathematical logic (even in the eight-year school), introducing many topics into the course (e.g., the study of differential and integral calculus in the eight-year school), and broadening and supplementing the study of a number of

topics (e.g., the principles of vector analysis, the principles of the theory of groups, etc.).

The most interesting thing is that both views can be justified, and the justifications appear outwardly conclusive.

On the wisdom of a 'return to Kiselev', for instance, one can invoke the need to simplify the school course by excluding a number of new ideas. One can point out that scientific knowledge rapidly becomes obsolescent, and that the future technician and engineer (the school child of today) will, in any case, have to cope with a different form of mathematics from the one he is learning at school. One can claim that the jobs done by highly skilled workers do not call for a knowledge of many of the things the course currently includes (e.g., the language of set-theory, the logical structure of geometry, etc.).

The further updating of mathematics, on the other hand, can be justified by the need to keep up with current developments in both mathematics and science, by the need to acquire the rudiments of mathematical logic as a result of the widespread use of computers, by the value of forging earlier and closer links between mathematics and science teaching (e.g., through derivatives, integrals and vectors), and by the fact that psychologists have demonstrated the child's ability to master mathematical abstractions earlier.

The resolution would appear to lie somewhere between these two extremes. One has to remember that education at school is both general and compulsory, while not forgetting the demands laid on education by scientific and technological progress.

There is a belief that, in discussing the structure and the content of the mathematics course of the future, it is necessary, first of all, to define one's criteria and to find the best solution to a number of educational problems.

Issues to be resolved

It is essential, for example, to give a clear answer to the following questions:

- (1) Should the logical arrangement and the main lines of development of the present mathematics course be changed? If so, in what directions, and why?
- (2) What should form the basis of the school mathematics course in the context of universal secondary education and of continuing scientific and technological progress? What kind of mathematical knowledge, know-how and experience should a pupil possess on completing eight years of education in order

to finish his secondary education successfully, whether at a secondary school, a technical college, a SPTU² or a SRM³, while at the same time receiving an appropriate vocational training?

- (3) To what extent can the power in children to reason be developed through the teaching of mathematics in the eight-year and secondary schools? The importance of this question is considerable, for it derives from the fact that (as shown by research done by Soviet psychologists and educationalists) the success of teaching is closely linked with the development in pupils of abilities to reason. This is because reasoning powers determine independence of judgement and an ability to pick out what is essential in the material under study. In contrast, the correlation between successful teaching and the degree of memory development, for instance, is less marked.
- (4) To what extent should the school mathematics course of the future be a sequel to the mathematics course in use today? What parts of the existing course should be retained in the future course? What changes should be made in the content and in the problems and exercises that are given to pupils?

Only when the principles have been clearly established will it be possible to draw up and seriously discuss alternative syllabuses, carry out local and large-scale trials and inform the public about the changes that can be made in the content of education.

It is necessary to go very deeply into what mathematical knowledge, what know-how and experience and what level of development in schoolchildren are important for their broad, general education, and what are needed for different forms of specialist education and for various kinds of work. It would be useful, in this connection, to analyse the mathematical content of the jobs done by workers, engineers and technicians employed in the country's foremost industries.

A possible way forward

We should like to set out our views on the principles to be applied in effecting further improvement of mathematics teaching in Soviet secondary education.

The modern mathematics course tends to be packed with new ideas, concepts and facts. Naturally, in the time available, not all pupils are capable of mastering such a great mass of material and memorizing such

2 SPTU = secondary vocational and technical college

3 SRM = school for young workers.

a plethora of definitions, theorems, formulae and properties (not to mention proofs). Any further renewal of the content of mathematics teaching must, therefore, avoid a substantial increase in the volume of mathematical knowledge required by arranging more carefully the conceptual structure of the course, and by improving the logical and methodological organization of the material.

In our view, one promising possibility for the improvement of the school mathematics course lies in '... organizing the relevant material on the basis of recapitulative ideas, principles, concepts and patterns specific to each science, making it possible at one and the same time to take in a large volume of factual material and to facilitate its study and the application of what has been learnt'.⁴

Another method of ensuring the productive assimilation of mathematical knowledge, but one which as yet is not fully implemented, is to differentiate between its constituent parts, selecting from the sum total the fundamental or most important items — i.e., those that must be mastered because they are *indispensable* for everyday life and work and vital for every educated person — and those items that play only an *auxiliary* role and assist the effective learning of the basic knowledge. In the content of today's mathematics teaching — especially now that there is universal secondary education — this is extremely important.

Criteria for content

Our research and our experience alike show that there are quite a few criteria which would enable the practising teacher to pick out the knowledge and skills that must be regarded as essential. For example:

- (1) If a part of the course is to qualify as essential, it should be both a means for the subsequent study of mathematics and a subject of study itself. Its importance can be judged from how far it is applicable to the solving of problems, the proving of theorems, the substantiation of principles, and so on. We shall call this criterion the 'instructional significance' criterion.

Judged by this criterion, for example, vector algebra is fundamental because it is an effective means of solving problems (and proving theorems) and, at the same time, a subject of actual study in connection with such problems.

- (2) Essential material should operate actively over a prolonged period (throughout the study of a topic, section or course).

Graphs of elementary functions, for instance, are used continuously all through the teaching of algebra to school children.

4 *Narodnoe obrazovanie*, 1972, No. 1, p. 121.

Teachers therefore devote considerable attention to graphical representation.

We shall call this the 'activity' criterion.

- (3) Essential material should permeate completely all three stages in a school education: knowledge, know-how and experience. An example of this might be computation with natural numbers.

This criterion, which makes it possible to select essential material that permeates completely the three stages of education, we shall call the criterion of 'completeness'.

- (4) Essential material should provide ample opportunities to inculcate in pupils a dialectical and materialistic world outlook. Studying such material should clearly demonstrate that mathematics, reflecting the forms and relationships of the material world, is a science concerned to develop mathematical models of concrete reality.

The concept of a derivative, for example, is one of the generalized concepts that embodies a diversity of processes which occur in the actual world. It thus has considerable significance for one's understanding of the world.

We shall call this the 'world outlook' criterion.

- (5) Essential material always constitutes a general statement about concrete facts that have been (or are being) learnt (such as the concept of transposition in geometry); this is the criterion of 'generality'.
- (6) Essential material should have a marked practical orientation, this criterion being one of 'applicability'. The concept of the derivative of a function, for instance, satisfies this criterion.
- (7) Essential material should be such as to make it possible to establish close links between and within subjects.

The concepts of a derivative and of an integral, for example, are used to arrive at formulae for the volumes of geometrical figures and are widely applied in physics and engineering.

We shall call this the criterion of 'breadth'.

- (8) Essential material must constitute an effective means of fostering in pupils the principles of dialectical reasoning.

It is legitimate, for instance, to introduce the concepts of variables, transformations, etc. into the school mathematics course at a fairly early stage, and to study contrasting concepts such as continuous and discontinuous functions, etc.

We shall call this the criterion of 'development'.

Ideally, of course, the 'essential material' should satisfy all the criteria simultaneously. In practice, this is not always possible, although there are some concepts (e.g., that of a derivative) that satisfy many of them. Clearly, it is precisely such concepts that should constitute the nucleus of the course.

When classifying the various topics in the syllabus as 'essential' or 'secondary', the same criteria should of course be extended to the problems, which, as a rule, appear in textbooks immediately following the material being studied. Those which are selected for solution first of all should be those that correspond in content to the syllabus items which belong to the essential group.

To turn mathematical knowledge into an effective means of education and a working instrument, and to use the learning of maths to develop pupils' reasoning and extend their education, enough time must be allowed in the curriculum for solving mathematical problems.

It is understandable why the establishment of scientifically-based criteria for identifying a body of fundamental mathematical knowledge, know-how and experience is one of the urgent tasks facing the theory and practice of mathematics methods today.

Summary

In this paper, we have touched on only some of the chief, practical considerations which apply to the further improvement of mathematics teaching in secondary schools. The basic working principles for improving the mathematics syllabuses and textbooks can, in our opinion, be summed up briefly as follows:

1. The improvement of mathematics syllabuses and textbooks for secondary education should be approached systematically.
2. The mathematics syllabus and textbooks should reflect the basic principles of education: the scientific character of education, its links with real life, its ability to train and to develop, to ensure a conscious and lasting assimilation of knowledge, order and continuity, accessibility and clarity. Particular efforts should be devoted to ensuring an inter-relationship between the principles of the scientific nature of education and its accessibility, and to making sure that all pupils master the fundamentals of mathematical knowledge.
3. The school mathematics course must be directed towards the systematic inculcation in pupils of a Marxist-Leninist world outlook, towards arousing an interest in mathematics and its applications, and towards developing the pupils' cognitive independence and reasoning powers.

4. Inherent in the general secondary school mathematics syllabus must be the possibility of making improvements in keeping with the pace of scientific and technological progress, while providing the mathematical training that pupils need at the present stage of our society's development. A body of fundamental mathematical knowledge, know-how and experience, such as will remain relatively stable over a considerable time, must be clearly defined in the syllabus. Other elements which are superimposed on that body of fundamental knowledge should be renewable as may be required to match changes in the mathematical training required by pupils at eight-year and secondary schools. At the stage now reached in preparing the syllabus and textbooks, particular attention should be paid to establishing a multidisciplinary approach to knowledge, and preparing pupils for work in present-day industry.
5. Work on the mathematics course for the eight-year school should be such as to make it possible for a secondary education to be obtained with equally good results in a school, a technical college, a SPTU or a SRM — together with appropriate vocational and pre-vocational training. Opportunities for pupils to study mathematics in greater depth and breadth should be provided primarily through club and optional activities, and other forms of extra-curricular work.
6. The school mathematics course should be constructed making the fullest possible use of links within and between subjects. There is a case for the wider use in the eight-year school of the constructive, algorithmic approach to the introduction of mathematical concepts, for relying, when presenting the material to be taught, principally upon the method of induction from specific examples, and for introducing the elements of deduction progressively, at a level of difficulty accessible to the pupil's comprehension. At the senior secondary-school level, in addition to the further linear study of mathematics, there should be a methodical ordering and consolidation of the material learnt in the eight-year school, and a deductive presentation of certain sections of the mathematics course.
7. The syllabus and textbooks should be so arranged that the study of theoretical material is directed towards its use in the solving of problems (educational and practical). Certain topics in the mathematics course can be incidentally revised in the course of solving a given type of problem. It is desirable that a large portion of the teaching time in each grade of secondary school should be devoted to working exercises and solving problems. In addition to solving problems which allow an algorithmic solution, problems of

a non-standard kind, that are within the pupil's capabilities, should also be set.

8. The mathematics syllabus and textbooks should enable the pupils to master specific mathematical procedures and skills (computation, identity transformations, geometrical construction, measurement, etc.) and also certain mathematical methods (coordinate, vector and geometric transformations, equations, etc.) at the level at which they are in fact used in the lessons in school.
9. Mathematical terms and symbols should be introduced progressively as the need for them arises. The terms and symbols introduced should be actively employed throughout the whole period of schooling, and should be consistent with the terms and symbols in common use in technical books. The concepts of set theory (and the symbols and terms associated with them) should appear primarily as a convenient language in the presentation of the school mathematics course.
10. The basic syllabus, and the syllabuses for optional activities, should make it possible to go on to the study at university level of those subjects that employ mathematical techniques.
11. In renewing and modifying the mathematics syllabus, care must be taken to ensure that the new syllabus follows logically on from the old.
12. Part of the preparation of the new maths syllabus and text-books must be to ensure that teachers are given instruction on the methods to be used in good time. To achieve this purpose, the teaching community must be effectively informed about the proposed changes in the course. The finished syllabus must be extensively and thoroughly scrutinized, and the trial textbooks must be adequately tested experimentally and corrected before being introduced in practice into the general schools.
13. The determination of the syllabus and textbooks for the entire secondary school mathematics course should be entrusted to a single team, which should be properly representative. It should include mathematicians, experts in teaching methods, staff from further training institutions for teachers and from bodies connected with education, as well as experienced teachers. Renewal work on the syllabus and the textbooks should be of an official nature. The right circumstances must be established for its success. Its progress should be monitored stage by stage.

Work on the further improvement of the existing mathematics syllabus for secondary schools is proceeding in line with the principles set out above. At the same time, ideas are being voiced about the

possibility of building up a syllabus that would be more positively oriented towards the application of mathematics in technology and the economy. Naturally, following the preparation of such a syllabus, there will be a great deal of work to do to produce experimental textbooks and try them out in schools.

TABLE 1

ANNEX

The allocations of teaching hours to the various topics in the algebra course for the sixth to eighth grades.

Number In syllabus order	Grade and Topic	Number of Hours	
		Under the existing programme	Under the planned programme
Grade VI			
1.	Basic concepts	17	18
2.	Functions	30	30
3.	Systems of linear equations	17	16
4.	Powers with natural indices	21	20
5.	Multinomials	41	40
	Recapitulation	14	16
TOTAL:		140	140
Grade VII			
	Recapitulation	—	4
6.	Rational fractions	40	36
7.	Inequalities	21	20
8.	Approximate computation	9	18
9.	Square roots and quadratic equations (together with quadratic trinomials)	44	32
	Recapitulation	8	12
TOTAL:		122	122
Grade VIII			
	Recapitulation	—	4
10.	Quadratic trinomials	—	10
11.	Equations and inequalities in two variables (together with equations reduced to quadratic equations)	14	15
	Approximate computation	10	—
12.	Arithmetical and geometrical progressions	20	15
13.	Powers with rational indices	26	30
14.	Exponential functions	14	12
15.	Decimal logarithms	35	24
16.	Details of computation systems and computer technology	3	6
	Recapitulation	18	24
TOTAL:		140	140

TABLE 2

Allocation of teaching hours to the different parts of the geometry course for the sixth to eight grades

Number In syllabus order	Grade and Topic	Number of Hours	
		Under the existing programme	Under the planned programme
Grade VI			
1.	Basic concepts	12	9
2.	Congruency of figures, and transposition	14	16
3.	Geometrical constructions	11	12
4.	Parallelism and parallel displacement	15	12
5.	Polygons	10	12
	Recapitulation	8	9
TOTAL:		70	70
Grade VII			
	Recapitulation	—	2
5.	Polygons (continued)	17	17
6.	Circumferences and circles	14	13
7.	Vectors	16	16
8.	Similarity	32	29
	Recapitulation	9	11
TOTAL:		88	88
Grade VIII			
	Recapitulation	—	2
9.	Turning points and trigonometrical functions	18	18
10.	Metrical correlations in triangles	8	10
11.	Inscribed and circumscribed polygons	20	20
12.	Introduction to three-dimensional geometry	16	11
	Recapitulation	8	9
TOTAL:		70	70

TABLE 3

Allocation of teaching hours to the different parts of the algebra and analytical principles course in the ninth and tenth grades

Number In syllabus order	Grade and Topic	Number of Hours	
		Under the existing programme	Under the planned programme
Grade IX			
	Recapitulation	—	6
1.	Principle of mathematical induction	5	—
2.	Elements of combinatorial analysis	11	—
3.	(1) Real numbers. Infinite series and their limits	20	20
4.	(2) Limits of functions, and derivatives	22	26
5.	(3) Application of derivatives	15	15
6.	(4) Trigonometric functions, their graphs and derivatives	23	26
	Recapitulation	9	12
TOTAL:		105	105
Grade X			
	Recapitulation	—	5
7.	(5) Trigonometric functions, their graphs and derivatives (continued)	34	34
8.	(6) Integrals	10	12
9.	(7) Exponential and logarithmic functions and their derivatives	20	22
10.	Systems of equations and inequalities	14	—
	Recapitulation	27	32
TOTAL:		105	105

TABLE 4

Allocation of teaching hours to the different parts of the geometry course in the ninth and tenth grades

In syllabus order	Number	Grade and Topic	Number of hours	
			Under the existing programme	Under the planned programme
Grade IX				
		Recapitulation of eight-year school course	—	4
1.		Fundamental principles of three-dimensional geometry. Parallelism in space	20	20
2.		Transformations in space. Vectors.	20	17
3.		Perpendicularity in space. Dihedral angles.	24	23
		Recapitulation	6	6
TOTAL:			70	70
Grade X				
		Recapitulation	—	3
4.		Method of co-ordinates in space	10	10
5.		Polyhedrons	22	20
6.		Figures of rotation	18	20
		Recapitulation	20	17
TOTAL:			70	70

The Continuing Mathematics Project in the United Kingdom

Many historians appear to avoid studying and interpreting recent events. It is difficult for them to achieve that necessary sense of perspective on evidence which is still partial and in a sense molten. The whole range of results of recent decisions and actions has yet to be seen. Experience encourages caution in making close analytical comment at an early stage.

Writers of case studies of curriculum development projects have to meet the need to share experiences with others in the field while heeding the warning illustrated by the trained historian. Enthusiasm for an enterprise, and, perhaps, even some local and immediate successes have to be seen in the context of the current educational system, and also in terms of likely developments of the work.

This study aims to provide an account of the Continuing Mathematics Project (CMP) in the context of current curriculum development activity in the United Kingdom, giving some attention to the problems which were encountered, and the inadequacies of its efforts, together with evidence of what was achieved. The views expressed are those of the author. They are not necessarily shared by the project team or by the project's sponsors.

The project provides an example of an attempt to grapple with the endemic, but diffuse, problem of students who, on passing out of the secondary into the tertiary level of education, possess an inadequate mathematical background for the successful pursuit of specialist study in subjects which use mathematics. Implementing the project involved a considerable investment of resources into a relatively uncharted area of the curriculum. And if the high aspirations of the project's originators were not entirely realized, some progress was made in understanding and meeting the needs of users of mathematics. In addition, important experience was gained which should add to knowledge of the potential (and of the pitfalls) of curriculum development projects.

Context

In the United Kingdom, curriculum decisions are normally made at the school level. While local authority advisers/inspectors and Her Majesty's Inspectors may make recommendations, give advice and even exert some pressure, the responsibility for the curriculum of each school is formally entrusted to its governing body; its implementation is the responsibility of the head teacher. In the secondary sector (11–18 years), however, external examinations, together with pupil and parent expectations, and, not insignificantly, teacher expertise, constitute constraints which inhibit radical variations, and it is an anomalous feature of curriculum planning that, while the system would seem to enable innovation at a local level and provide the possibility of a mass movement within the system, it more normally appears as a damping medium for the implementation of innovation. Even the strikingly rapid growth of the use of School Mathematics Project materials in the United Kingdom does not match the immediate universal introduction of a new curriculum and its materials which is evident in some other countries.

While some curriculum development projects have started with minimal funding in a single school or within a group of institutions, and while others have received fairly substantial funding from industry (e.g. Nuffield Science, Mathematics and Language), the largest single sponsor of curriculum development in the United Kingdom has been the Schools Council. Established by the Department of Education and Science in 1964, the Schools Council was given the responsibility of co-ordinating secondary school examinations, and assumed the task of encouraging the funding curriculum development. Since it was established, over one hundred projects have been funded; some on a very small scale, others with budgets in excess of £200,000. Some thirteen of these projects have been in the field of mathematics, covering the age range 3–18.

The structure of secondary education has been transformed during the last twenty years by three developments: the evolution of a comprehensive system of education; the raising of the school leaving age to 16; and the introduction of a national system of examinations for those whose ability at age 16 falls roughly between the 20th and the 60th percentiles. Nevertheless, the structure of education for 16 to 18-year-olds – the so called 'sixth form' – has its roots in the former selective system. In the 'grammar schools' which previously catered for the most able 25 per cent of pupils, it was customary to admit students who had achieved reasonable success during a five-year course of general education into a specialized two-year course, during which they would concentrate on three or four subjects in which they would be examined

at 'Advanced Level' at the age of about 18. These arrangements continue to obtain in the new 'comprehensive schools'. And the results of the 'Advanced Level' examinations are important determinants of eligibility and selection for higher education. The system enables school teachers to engage in intellectually challenging work, and it has allowed universities to assume a sufficiently substantial basis of knowledge to make provision for courses leading to an honours degree in three years. It has also provided able students, still at school, with a somewhat narrow, but relatively advanced course in self-selected fields. While many groups have jealously guarded the virtues of the 'sixth form' system,¹ others have questioned the desirability of such narrow courses at the school level. (The United Kingdom appears to be unique in deciding that such a pattern is appropriate.) In particular, it is questioned whether such courses are appropriate for those students who are not intending to go on to higher education. The debate has been in progress for at least fifteen years, but the arguments for change have been strengthened by the developments in secondary education which were described earlier. However, even if the current proposals² to broaden the sixth form curriculum are adopted, they seem unlikely to be implemented for at least eight years.

Origins of the Continuing Mathematics Project

The description of the 'sixth form' system has been drawn in some detail because it is an important prerequisite for an understanding of the origins of the project and of its subsequent work in schools. In 1968, Sir Frederick Dainton's report³ on the flow of candidates in science and technology into higher education appeared. It recommended that all pupils remaining at school for sixth form work should continue to study mathematics. While, as a general principle, there might have been broad assent to the idea, there were a number of immediate practical difficulties. How would such work be fitted into the students' timetables? Who would teach the additional courses when

- 1 Some indication of the effect of this system is seen in the performance of the British team in the International Mathematical Olympiad for which little particular preparation is given. Also in the International Association for the Evaluation of Educational Achievement (IEA) study of mathematical achievement in twelve countries where the relatively strong performance of United Kingdom students at the age of 18 is in sharp contrast with the moderate results at 13 [T. Husén (ed.) *International Study of Achievement in Mathematics*, Vol. II, p. 22-5, 1967]
- 2 The 'Normal' and 'Further' level proposals envisage a curriculum involving the study of five subjects.
- 3 *An enquiry into the flow of candidates in Science and Technology into Higher Education*, HMSO, 1968.

there was already a national shortage of suitably qualified mathematics teachers? What mathematics could one justify as relevant to a group of students with such diverse interests?

Developments elsewhere in the educational system provided not only the room for manoeuvre, but also the impetus for at least a partial answer to be attempted. While students intent on taking physical science courses were probably already studying mathematics to an 'Advanced Level', those interested in the biological and human sciences were unlikely to continue their work in mathematics beyond the age of 16. Yet their main subjects were increasingly adopting a quantitative approach at university level, and it was clear that quantitative work would find its way into school courses within a few years. The needs of the life and social sciences presented an immediate focus for identifying the type of mathematics which would be relevant to the studies of students who were not taking courses in physical science.

In the late 1960s the National Council for Education Technology (NCET) was established to explore and to encourage the use of educational technology within the educational system. In the light of the Dainton Report, it commissioned A.G. Howson and M.R. Eraut to examine the feasibility of launching a project to demonstrate a systems approach to providing for the mathematical needs of sixth-formers who were not specializing in the subject at 'Advanced Level'. Their conclusions⁴, submitted in 1968, recommended the establishment of a four-year development project which would prepare largely self-instructional materials.

Although NCET was not so funded as to enable it to sponsor such a project on its own, it was able to provide the initiative and drive required to raise the necessary funds. In particular, about a quarter of the budget was raised as a result of approaches to a number of industrial companies. Substantial contributions were also made by the Department of Education and Science, by the Scottish Education Department and by NCET itself. But the biggest share, 40 per cent of the necessary finance, was provided by the Schools Council who also undertook responsibility for any later follow-up work.

Accordingly, a steering committee consisting of representatives of donors and others was brought together, and it was agreed that the work of the project should be based at the University of Sussex, where there were particular expertise and resources relating to educational technology.

⁴ Howson, A.G.; M.R. Eraut, 'Continuing Mathematics', NCET Working Paper No. 2, 1969.

Evolution of the project

In 1971, the project started its work with a nucleus of a team that was gradually built up over a period of two years. At a later stage, the development work was extended for a further (fifth) year, and additional funding from the Schools Council and the Council for Educational Technology (CET) allowed a small team to implement a dissemination phase between 1976 and 1978.

Initial planning had recommended the development of a core course for all students in need of further mathematics, together with branching programmes of work for those with particular subject needs. Material for the core course in both statistics and calculus was accordingly prepared and tried in a number of schools. This experience quickly brought into focus the problems of context, of variable student background and motivation and of the teacher's uncertainty about his role when using self-instructional materials. Further, it stimulated discussion of the mathematical needs of the sixth-formers as these were perceived by the mathematics and science teachers in the trial schools. As a result, some two years after the project had started its work, a new model for production was established. The new design was able to accommodate much of the early writing, but the more advanced work which had been anticipated had to be sacrificed to make room for the inclusion of revision material in basic mathematical skills and introductory 'units' of material on relatively recent applications of mathematics. A 'unit' of work was conceived to be an assignment which would occupy the student for about two to three hours. While the ingredients of a unit varied considerably, the length of time seemed appropriate for students of this age and level of maturity. Although few students would complete a unit in one sitting, it did, in the event, prove to be a practical guide for unit length.

The eventual structure of the material produced by the project consisted of three 'categories' of units. An indication of the characteristics of units within each category, together with some examples of units which were prepared in each of the categories, are given in Figure 1.

Form of the material

The feasibility study had stressed the need for the project to explore the use of a variety of media. It became, however, standard practice to prepare for each unit a text providing instruction in a programmed form. Student responses to the text were required fairly frequently in various forms: written answers to questions posed, completion of tables, completion of graphs, and the solution of problems. Additional

Figure 1.

Category	Purpose	Characteristics	Examples
1.	Development of basic skills for use in main subject and/or for work in Category 3.	Single independent units of a background or remedial nature. Direct and fairly concise development of material	Indices and Standard Form. Working with Ratios. Inequalities. Equation of straight line. Theory of Logarithms.
2.	Designed to motivate student who has no well-defined need for particular mathematics by (i) selection of topic and (ii) style of presentation.	Single independent units of an introductory nature. Take a broad view of topic with minimal demands on pre-requisite knowledge and low priority for skill and technique acquisition.	Critical Path Analysis. Flow Charts. Systems. Information and Coding. Linear Programming. Mathematics of Finance.
3.	Development of skills and techniques together with understanding.	Sequences of units providing a systematic development to the level required by the students. Some extrinsic motivation expected on part of student. Some units for particular subject needs (e.g. Biology, Geography).	Descriptive Statistics. (4 units) Probability (3 units) Hypothesis Testing. (3 units). Calculus (6 units). Mathematics for Geographers. (4 units). Mathematics for Biologists. (2 units).

exercises (with detailed solutions available) were provided to encourage a grasp of the whole, a grasp which can easily be lost in following the steps of a programmed text. The unit concluded with a 'summary', which the student was encouraged to read before attempting the 'post test'. A copy of the summary could then be retained by the student for reference and for revision.

In exploring the use of media other than textual material, the project had to bear in mind the implications of using expensive material, even if it were shown to be effective. The rapid growth in the use by schools of video equipment encouraged the exploration of this medium in the latter stages of the project. The available finance, however, was insufficient to match the quality of production to which a generation of students raised on television are accustomed. One suspects that if the project had been born a few years later, it would have entertained the prospect of producing videotapes with more conviction.

The audio-visual components of the material were consequently of a form which was simple and relatively cheap both to produce and to distribute — that of film-strip and tape. Early tape/film-strips provided an introduction to topics through their applications. Their aims were to motivate interest and provoke thought. Trial, however, revealed a contrast between the aspirations of the project and the students' expectation of mathematics. While teachers often appeared to appreciate the 'motivating' approach, students generally regarded it as irrelevant and a distraction from the task in hand. A second style of tape/film-strip, which has now been developed to the stage of publishing, adopts a more didactic approach. This sometimes takes a student through a section of work so as to give him some idea of the 'whole' before he makes a detailed study of the methods and techniques involved. Elsewhere, the audio/visual component is employed when there is a need to discuss the shapes of distributions, areas under curves, etc.

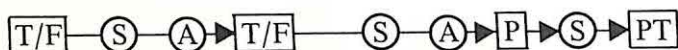
Other styles of media which were developed by the project and used during the trials included apparatus. Examples are 'the gradient measurer' in the Calculus course, experiments for the statistical units, and games in units on Critical Path Analysis and Flow Charts. The financial implications for users of making such materials available at the publication stage required strong evidence of the value of their contribution to the students' learning. Such hard evidence is difficult to obtain. The materials which eventually emerged represent a compromise: the apparatus was left out; the games and the experiments are provided in printed form, but in a manner which does not enhance their chances of actually being used. Apparatus for the experiments which are recommended has to be assembled by the student or the school. Materials for the games — for instance, sets of cards — have to be prepared from

printed sheets of thin paper. The additional labour involved in the making of these materials appears likely to dissuade the majority of students from ever experiencing them.

Some idea of the way in which the various components of a unit were assembled, and how a number of units were linked in order to construct a short course can be obtained by considering the four-unit sequence on 'Descriptive Statistics' shown in Figure 2.

Figure 2

Unit 1 Presenting Statistics



Unit 2 Modes and Medians



Unit 3 Mean and Standard Deviation



Unit 4 Some Experiments

Key:	T/F	Tape/film-strip
	S	Summary
	A	Assignment
	P	Programmed text
	PT	Post test

Rather more than fifty units were eventually produced by the project. Of these, some sixteen were revision units in Category 1, six were introductory units in Category 2, and the remainder in Category 3. By the winter of 1978, forty-four of the units were to be in published form.

The style of publication was largely determined by the Schools Council's wish to make the materials available to as wide an audience as possible during a time of economic difficulty for the country. In the period 1974-6, when a publisher was being sought, publishers were not attracted by the complexity of the materials, seeing that they were intended for a diffuse market in schools, in colleges and in higher education, where the departmental target was apparently large, but where it could not be guaranteed that individual departments, with their scarce resources, would respond avidly to marginal course materials.

A bid, however, was accepted from a major educational publisher, Longmans Ltd. This firm had established a special production unit in York in order to provide resource material in printed form relatively inexpensively. Booklets of A4 size, with up to thirty-six pages, can be prepared at York by an offset litho process using 'camera ready' copy. By this method of production, project units were marketed at the attractive price of 40p per copy. But the constraints imposed by the form of publishing were in conflict with many of the features of the learning system which had been developed. In particular, 'answer booklets' could not be provided. The unit summary, the post test and the answer sheets to the post test, together with games — items which were designed to be provided separately — all had to be absorbed into the single text, along with notes for the tutor. The result is a uniform set of units which, by their form, do not provide the explicit support for the learning system which was developed during the trials. The mode of publishing uses central storage and distribution. This ensures a rapid despatch service and an ability to reprint at short notice. However, it does rule out the possibility of casual purchasers obtaining single copies of units at their local bookshop. On the other hand, there is one considerable advantage in this method of distribution. It provides the opportunity for knowing who has purchased project material. Such information is clearly invaluable in any study of dissemination.

Development of the materials

Establishing the team

The authors of the original feasibility study recognized that the preparation of self-instructional materials would involve a considerable investment of time, particularly where some of the materials would be in audio-visual or in other non-textual format. During the development phase of the project, as many as nine people were involved in writing material for periods ranging from one to four years. All the writers had experience of teaching in secondary schools. Eight of them were mathematicians. One was a specialist in the audio-visual aspects of educational technology who happened to be a geographer. As well as writers, the project had the regular services of a part-time graphics artist, and three secretaries. Advice and help was provided by a number of consultants, and, throughout most of the project's life, a succession of British Petroleum Company Ltd. (BP) Schoolmaster Fellows at Keble College, Oxford, were linked with the project. Several units of material were prepared in draft form by 'outside writers'. These were interested teachers and lecturers who were commissioned to prepare particular units.

Writing the material

Each of the project's units had to pass through a sequence of steps, from the original decision to commission a unit to the final submission of the unit in camera ready copy to the publisher, a process which seldom took less than two years. Since the material was designed to be used by students working individually, the instruction had to be detailed, and it inevitably reflected the teaching style of the author. Writers were confronted by the dilemma of having to expose their styles of teaching to academic critics, while being conscious of the need to use various methods — sometimes unorthodox — to communicate with the students. Detailed editing at the draft stage would have been difficult once the production line began to move. And, since the project team had to manage its affairs for a year without a director, it established a moderation procedure to provide individual authors with advice and to ensure some control of the form of the material.

Usually two members of the team took responsibility for commenting in detail on a colleague's work. In addition, units were examined by one or, more commonly, by two external moderators, one a teacher familiar with the target audience, the other someone with particular expertise in the subject matter. For a small fee, these moderators were invited to comment on the units according to a specification prepared by the evaluator. The comments received were often helpful and illuminating. In some cases they led to substantial improvements in the units. On occasions their views were at odds with those of the author, and the director was required to arbitrate. However, after team members had passed through the initial stages of extreme sensitivity to criticism of their work, they were able to value the comments and the discussion, which became the normal way of proceeding.

A second means of refining an author's work was to invite students to express their opinion. This formed one element in a two-part trials strategy. First, it was hoped to improve the efficacy of units by soliciting the comments of students and teachers on the material, and by examining the students' responses to the questions put in the text and in the post test. Second, there was a need to discover how the material could be used effectively in a variety of schools and colleges. While some students were articulate and most of them were co-operative, their comments were, on the whole, better pointers to the problems than steps towards their solutions. However, their reactions did enable team members to identify problem areas and prompt them to try alternative approaches. With the early units, it was possible to put some through two complete testing cycles if it seemed necessary.

The plan for testing the project's materials involved three stages, and a gradual shift of interest from the trial of individual units to the

operation of the learning system as a whole. During the first stage, a unit would be used by a small group of students with the author at hand to discuss obscurities as they arose. Arrangements were also made for small groups of students to work through the same unit in the presence of two other members of the writing team. They, for their part, were subsequently able to provide the author with comments from their rather more objective standpoint.

The second stage of trials, termed 'field testing', enabled a larger number (twelve to twenty) of schools and colleges to use revised units of their choosing in courses which were supervised by teachers. At this stage, most units were used by over a hundred students, and the answer booklets returned by them, together with comment forms from tutors, not only provided evidence of student performance and of deficiencies still remaining within the unit, but also illustrated the variety of contexts in which the material was being used. This led to some informal hypotheses on the effects of such variables as curriculum context, the subject specialism of students, the subject background and the role of the tutor, different types of unit and the effects of using individualised material within substantial courses.

The third stage of trials involved almost one hundred institutions which were able to select units appropriate for their students' needs from some fifty units which had become available. While secondary schools formed the majority of the hundred institutions, the total also

Figure 3

Timetable context	Tutor	Method of working	
		Individuals working alongside other students in a classroom.	Individuals working independently in a library, resource centre or at home.
Non-specialist subject time (General studies)	Mathematician		
	Non-mathematician		
Specialist subject time (e.g. as part of a biology course)	Non-mathematician		

included sixth-form colleges, colleges of further education, polytechnics and a few universities.

The experience of field trials over a period of two years enabled members of the team to discuss matters concerning the choice of units and organization for individual study with teachers so engaged. It was clear that circumstances and needs differed greatly among the institutions taking part in trials. As a result, the materials were used in a wide variety of ways and for varying periods of time. In some cases, the units formed the whole course. In most cases, the units were embedded in courses designed by the teacher. Some indication of the modes of use within sixth-form work is provided by the matrix shown in Figure 3.

The evaluation activity was intended to be of a formative kind – gathering information for use by the team in improving units and in order to understand better the context of use. Figure 4 is an attempt to depict the overall structure of the trials strategy, and to convey some idea of the way in which data was collected.

Results of trials

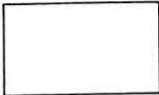
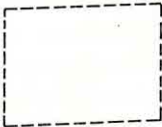
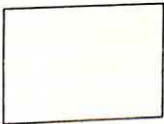
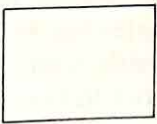
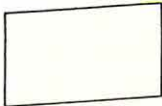
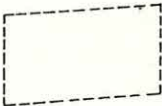
Many of the early units prepared by the project were drastically modified or abandoned after the first and second stage of trials. While teachers rarely condemned them, the response of students indicated that they were too academic, even though the gist of the criticism was that they were not direct enough in their teaching. Later units proved to be more digestible.

The first two stages of testing enabled writers to experiment with modes of presentation and to obtain helpful feedback. At each stage the students' performance on the post tests was closely scrutinized as a measure of the unit's effectiveness. While errors could be corrected, and greater clarity achieved, a substantial part of the unit was a product of the individual author's style and his own view of mathematics. Both **are difficult to modify substantially without destroying the work or its** author! In the event, the final editing of the units, carried out in part to meet the constraints of publishing, provided a uniform gauge through which all units had to pass. While, as a result, certain extremities of style were brought under control, final editing also led inevitably to a form of unit different from that in which the unit had been tested.

The trials brought into focus the difficulty of working across subject boundaries. While some mathematics departments jealously guarded the new project, others seemed almost unaware that their science colleagues were experimenting with CMP materials in order to provide mathe-

Figure 4

Organization of Trials

	<i>Development Testing</i>	<i>Field Testing</i>	<i>Validation Testing</i>
Students	10–20 students (3/4 local schools)	100–200 students (12/15 institutions)	Approx. 85 institutions.
Development of units			
Methods of data collection	Observation by unit authors and colleagues. Students' written work and comments. Comments by teachers.	Written work in student answer booklet. Feedback sheets. Tutor's comment sheet. Discussion between evaluator, tutor and students.	Student comments and post test performance. Tutor's comments.
Understanding the context of use			
Methods of gathering data	Discussion with members of staff at the school.	Institution Profile. Tutor's Record Sheet. Course Questionnaires. Discussion with tutors and staff in schools. Tutors' conference.	As for field testing.
Key:	 Main interest  Subsidiary interest		

mathematical support for science courses. In several schools a more constructive long-term approach was evident, with mathematicians acting as the tutor to the group of, say, biologists which included the biology teacher herself. In subsequent years the biology teacher became the tutor. Teachers were divided on the question whether non-mathematicians should supervise the work. Usually mathematics teachers were emphatic that the mathematical questions which students would raise could not be adequately handled by their science colleagues. However, scientists who had acted as tutors were more confident, and seldom felt the need to refer students' questions to the mathematics department, whose role was reduced to acting as backstop. The dichotomy of views, which the respective sides might interpret as overpossessiveness on the part of mathematicians and over-confidence on the part of scientists, reveals one factor in the situation which prevents mathematicians and scientists working together in any creative way. Despite the above, the project's materials were the means of leading a number of departments to consider how they might best provide for their students.

The varied contexts of use cautioned against laying down a rigid form of use. Nevertheless, problems of use were inevitable where teachers were unaccustomed to individualized learning, and it became necessary to identify a few general principles which would prevent some outcomes which were initially common. Problems involving the management of resources, in particular the audio-visual material, the timetabling of work and the pace of work, could largely be overcome with some forethought and the application of the team's experience to the circumstances of a particular school or college.

Some of the undermining features which were evident are illustrated by the list which follows:

- (i) Students sit in desks facing the teacher who is seated at his desk at the front as if for a formal teaching lesson.
- (ii) The teacher acts as guardian of the materials and their use — a barrier rather than a means of access.
- (iii) The study sessions are inappropriately matched with the learning material.
- (iv) The mode of use of the audio-visual components cause compromises for faster and slower pupils.
- (v) The teacher feels detached from the course he is tutoring. This has the effect of making him feel uneasy, if not dissatisfied.
- (vi) The students do not take responsibility for their learning, but appear to be doing their duty by presenting themselves at the class.

- (vii) There are no direct links with the subject which is said to require the mathematics currently being studied.

Teachers are usually so familiar with the context in which they are working that they do not consciously consider how they instinctively adapt their content and their methodology so as to minimize the effect of some parallel problems which arise when they are teaching. The introduction of 'foreign' material without the interpretation which it is usual to give when most textbooks are used can leave the teacher stranded unless he is committed to using resources, and has the will to tackle some of the immediate discontinuities caused. Some of the themes which came to be regarded as significant were as follows:

- (i) Students must be encouraged to take responsibility for their own learning. (How can this be achieved given this particular institution/timetable/subject context? If students are unfamiliar with such an approach, how might they be helped to adopt it?)
- (ii) Teachers have to create a tutor role in which they can feel at home, and in which they can use their skills to the greatest advantage. (What is the tutor's role? How is his time to be spent? How does his contribution relate to that of the material? How will students come to understand his role?)
- (iii) If the work is designed as a service course then the work must relate closely to the subject which requires mathematics. (To what extent is this possible with external, pre-prepared material? What additional material may be required? What other forms of linking are possible, e.g. timing, shared teaching, taught lessons, etc.?)
- (iv) The organization of materials with regard to both storage and use must be planned in order to ensure ease of access and to minimize staff time absorbed. (Is a simple resource system required for the course or can a departmental or school resource centre be used?)

Experience suggests that the key to the satisfactory use of CMP material can be found where efforts are made to face up to these questions. The units have been prepared with care, with a structure designed to encourage a systematic approach to learning. However, unless this internal structure is made to fit into the tutorial arrangements and the requirements of the main course, there is little hope of satisfaction for the tutor or success for the students.

Dissemination

In the early 1970s the experience gained by the Schools Council in the

field of curriculum development showed that 'dissemination' required special measures. A working party set up by the Council suggested that dissemination could be regarded as successful 'when teachers understand the project's ideas and materials sufficiently well to use them in school if they choose to do so'. Accordingly, the funds earmarked for new projects subsequently included an element for dissemination. Other current projects which were still active at the time were encouraged to consider how dissemination could be effected, and what additional resources might be required. CMPs proposals for a two-year dissemination phase called for the services of two persons and a secretary. These proposals were accepted.

During the dissemination phase, the team was occupied with two major strands of activity. First, editing later units and preparing them for publication. Second, ensuring that the existence of the project and its materials were brought to the attention of all appropriate branches of the education system.

The enormity of the potential market encouraged the director, in applying the dissemination strategy, to aim at breadth rather than depth. Not only were all mathematics, science, geography and economics departments in schools to be informed, but so also were their counterparts in sixth-form colleges, colleges of further education, polytechnics and universities. In addition, industrial and commercial training boards, and the armed services provided further areas where the material was likely to be of value. The route to the maintained sector of schools and colleges was seen to be through the local authority advisers and inspectors. Talks or short workshops were offered, and many such courses took place with a variety of audiences: mathematicians only in some centres; users in others; and just occasionally, a mixture which enabled a more substantial exploration of issues. The one-day workshop was, perhaps, the most satisfactory from the team's point of view. It allowed an introduction to the work of the project to be followed by the concentrated study of one unit. Where appropriate — for example, if the unit on the Chi-squared Test of Significance was studied — the study would be followed by an application of some data collected during the session. Teachers were then able to reflect upon their own reaction to the experience of self-instruction and judge if the method might prove appropriate for their students. The second part of the workshop allowed teachers to examine the range of materials produced by the project, and to hear of the way in which teachers are using it in their own schools. A first-hand report by a local teacher was always welcomed by other teachers since he was seen not to be selling his own wares. Workshops concluded with an opportunity for teachers present to react to what they had seen and done and to ask questions, in the light of their knowledge of the material and of their own situation.

Ideally such a workshop was seen as an introduction which allowed initial decisions to be made. There then follows a need for later meetings to allow those planning the introduction of such individualized work, or those actually engaged in it, to discuss the problems arising in their work.

The project has inevitably used a variety of other means to explain itself. Articles have been placed in various professional journals and exhibitions, and workshops have been organized at local and national conferences of science and mathematics teachers. Two years is clearly an inadequate period of time to complete such a task, let alone foster a continuing interest in developing new materials. Two groups were formed in different parts of the country to work on matters which were of particular concern. One group concentrated on diagnostic testing as a prerequisite for planning courses for students. The other worked on clarifying the specific needs of geographers. While both groups met a number of times, the members had neither the time nor the resources to develop their ideas to a stage where they could be made more widely available.

With the end of the dissemination phase in February 1979, an 'after-care' service will be provided to ensure that information and advice is available to individuals and institutions who may require it.

Reflections on the Project

The problem of catering for 16 to 19 year-olds who are insufficiently numerate to continue satisfactorily their studies in subjects which require mathematics is not a new one. The project, however, was a new attempt to find a solution. It was, moreover, launched at a time when changes were being made in the amount and type of mathematics required, and when there was an opportunity to explore the potential of educational technology in an area where there was a shortage of specialist teachers. What has been achieved?

Perhaps, as with politics, curriculum development is concerned with achieving what is possible. This doesn't stop those in education from aspiring to achieve what they would like to see as an ideal, but it may suggest a need for caution in what they say about outcomes. CMP has not transformed the scene. And is unlikely to do so in the long run, unless it catalyses a great deal more effort in the problem area. What it has done is to prepare a substantial number of booklets which purport to teach usable mathematics at and beyond the level normally reached by the more able 16 year-old pupil. The booklets are so written that motivated pupils can learn mathematics with minimal assistance from a teacher. They provide a resource for teachers — mathematicians or

users of mathematics — who wish to supplement their own courses. They are a major resource for teachers wishing to develop individualized work programmes in mathematics. (Some of the most striking work observed during trials occurred when CMP materials were being used as one part — often a major part — of a collection of resources, some prepared by the institution, others purchased.) In particular, the project identified some of the significant factors which inhibit progress in this area of mathematics education. It is hoped that this evidence may be of use to others who will undoubtedly tackle this problem in the future.

To conclude this study of an example of curriculum development which, even in the eye of its creators, achieved only moderate success, it may be worthwhile to explore some of the reasons for this.

Enquire about someone's health and you are more likely to hear of a cold than of cancer. While the former is prevalent and familiar, it is the latter for which we are anxious to find a cure. While the analogy is far from perfect, the problem of insufficient mathematical knowledge is a nuisance, but is something that teachers appear prepared to live with. If someone has a cure, then it will be readily accepted, but resources, in the form of time and money, are not available when they are urgently needed for main-stream examination-bound activities within a department. So, at present, most students either take little in the way of service mathematics for their biology, geography or economics, or they do a great deal, as when they follow a full examination course in mathematics, to 'additional' or 'advanced' level.

A major change in curriculum structure would create space for mathematics for all those who need it. However, significant progress can be made where sixth-forms and students come to their studies with less confined expectations on methods of learning. The experience of the project showed that sixth-form college students and tertiary students, who were starting a new stage in their education, expected change in the mode of learning and were better able to benefit from the materials.

If the curriculum environment was not ideal, how well suited was the team for its task?

Recruiting writing teams for short-term projects is a risky undertaking, both for the sponsors of the project and for the individuals employed. While subject knowledge and teaching experience are necessary prerequisites, many of the tasks which team members have to undertake will be novel to them. Little evidence may be available to guarantee success. While this could be said of many tasks undertaken in education, the curriculum project, unlike an institution, hasn't the

momentum to maintain a steady course. It has the stability of a canoe rather than that of a cruiser. In fact, the CMP team combined a considerable range of talents — many of which were unknown to those responsible for appointing them. The strategy of appointing mathematicians who would consult users to gain an understanding of their needs, has resulted in units which often adopt the viewpoint of a mathematician rather than that of the users. In retrospect, it was a pity that a user — perhaps a biologist — was not appointed as a full team member to inspire the team to wrestle more determinedly with the dilemma of balancing mathematical soundness against the pressing need to provide a usable tool. Further, all curriculum teams operate under some area of floodlighting — a situation which has its merits, but which does little to strengthen the confidence or encourage the imagination of those at work. There would seem to be strong arguments for entertaining in the curriculum field the possibility of small beginnings, which will either blossom into promising growth, or simply die quietly.

It is probably true to say that the project tried to tackle too much in too short a time. It cast its net wide, gathering in a diffuse set of problems which had mathematical needs as a common feature, but which also had all sorts of important, particular features which would defy any common solution. Even within the narrower confines of mathematical content such a phenomenon is evident. Mention 'handling algebraic formulae', and users on all sides will declare an interest. However, it would be naive to assume that a two-hour unit will sort out the multiple confusions that give rise to the multiplicity of problems which the users corporately have in mind. Concentration on the needs of a particular kind of user might have led to exemplary materials of use to other users. Certainly it would seem desirable for any future effort to be focused on a particular user, even if the needs of other users have to be temporarily left on one side.

Individualized learning requires the commitment of the teacher just as much as any other mode of learning does. But it requires considerably more, if the change of role for the teacher is to be welcomed and used creatively. Without such prerequisites, the experience is doomed to frustration or to a blasé indifference to what is actually happening to the students. Yet there is evidence to show that some teachers not only gain personal satisfaction, but also ensure that sound learning takes place with individualized materials. CMP has produced a significant addition to the resources which are available to such teachers. Its materials might also be the means by which some teachers begin to explore their capacity for encouraging students to be more independent in their learning.

Secondary Mathematics in the United Republic of Tanzania

Historical Note

It is a well known fact that mathematics has occupied a conspicuous place in the curricula of schools in many countries. Some have included mathematics as a matter of course; others have sought philosophical, psychological, pedagogical and many other justifications for doing so. The Greek philosopher Plato gives an account of the training of the philosopher king in his treatise *The Republic*¹. The training includes mathematics. Plato's reasons for including mathematics are probably different from those which curriculum developers in mathematics would espouse today. They are, nevertheless, perfectly plausible. Some countries have emphasized the 'training of the mind', a benefit which learning mathematics is said to bestow, and which has implications for the transfer of learning to other spheres of human activity. There are those which have stressed the utilitarian role, which emphasizes the practical use to which man can put his mathematical knowledge. Between these two views, there are many different shades of belief, and the United Republic of Tanzania's reasons for teaching mathematics at all levels of schooling belong there. Her reasons have also been coloured by the historical events which have shaped the nation-state of today.

The activities that pervade a State are a reflection of the nature of the society in that State. A free society, whose members can make their own decisions, will act with greater flexibility than will those under authoritarian regimes. Societies which have made great scientific and technological advances will design the curriculum for its young generation differently from those which have a long way to go. Nations which trade a great deal with probably place a greater emphasis upon commercial mathematics, such as accounting, than those with undeveloped commercial enterprises. The colonized will take what is given to them by the colonizer, and, in most instances, what is given will reflect the interests of the colonizer. In the newly-independent countries, there will be tendencies to hang on to what was bequeathed

¹ Plato *The Republic*. (Trans. H.D. Lee) The Penguin Classics, Part VIII. 1955.

to them by their immediate past masters — syllabuses, textbooks, jargon, the whole lot. There will be a tendency to look down upon indigenous knowledge as being of no consequence since efforts will previously have been made to relegate such knowledge to a lowly position in the formal education system. Where then does the United Republic of Tanzania stand in this matter?

The teaching of mathematics in today's United Republic of Tanzania reflects the periods of political and socio-cultural upheavals through which the country has passed. There is the strong impact of the Arabic influence on trade and language. The Arab slave trader, ruler or religious missionary introduced the Arabic mathematical culture and language both on the coast as well as on the routes through the hinterland along which his caravans passed. Many mathematical terms in Kiswahili, such as the measures of length and capacity, the measures of time and those of the calendar attest to this influence. Then there was the Asian trader who set up small shops at trading centres throughout East Africa, whose 'business mathematics' has made such a mark that it is now part of the country's culture. Mathematics is not just that which is taught in the classroom from a prescribed syllabus and the accompanying textbooks. It includes what is learnt in the practice of daily life in the different sectors of human activity. It could even be argued that it was through the influence of the Asian in East Africa that 'commercial subjects' in the former 'African' secondary schools were introduced. Official reports² indicate that while the 'African' secondary school was teaching the conventional 'grammar school' type subjects, the Asian secondary school, which was invariably situated in a town, stressed 'commercial subjects' — in addition of course to other subjects. Today, many existing private secretarial and commercial schools have an Asian origin.

When the German colonizer followed in the wake of the missionary, he introduced formal schools in which he taught mathematics plus other subjects. The primary aim of opening formal schools for African children was to prepare a cadre which would serve in the colonial service, in the plantations opened by the farmers, and in the business houses of the German merchant, who brought in manufactured goods and sent away raw materials produced by African labour on German-owned plantations. Due credit must be given to the Christian missionaries who pioneered in formal education, although they too had a purpose. This was mainly to convert, to serve the converted and to maintain the missionary enterprise. The type of mathematics taught reflected therefore the needs of the German and missionary interests.

² Government of Tanganyika. *Annual Reports of the Department of Education*, Dar es Salaam, United Republic of Tanzania, different years.

The League of Nations, which was formed after World War I, placed the former German colony, which is today's Tanzania mainland, under the British as a mandated territory. Overnight, the language of instruction changed from German to English. The metric system of measures, introduced by the Germans, was abandoned and replaced by the 'Imperial units' – the 'furlongs', the 'stones', and the 'farthings', the 'half-crowns', etc. In a matter of a few decades, therefore, the Tanzanian man or woman had absorbed Arabic measures, Asian measures, German measures and British measures. That he took all this in his stride must be a matter of pride for himself and of wonder to those who fail to cope with a single set of measures in one language.

The British aims of education, as dictated by the League of Nations, were to prepare the people for eventual self-determination. In practice, these aims included the training of a loyal civil service, and creating a pool of skilled workers for the business interests and for the farming community. It is no wonder therefore that a British-type curriculum was prescribed, and that pupils were prepared for English examinations. Anyone who went through the system must remember the many hours spend in grappling with problems on journeys in the underground from tube station X to tube station Y, problems on the life expectancy of a coalminer in Newcastle, and the results of a cricket match in a county in Southern England, etc. These points are being made so as to place in context later observations on how today's efforts have been directed towards making the teaching of mathematics more meaningful to Tanzanian youth.

Apart from the trader, the missionary and the colonizer, other influences can be traced to the powerful multinational corporations operating in this part of the world. Boys and girls are aware of the possibility of obtaining attractive posts as accountants, quantity surveyors, bankers, etc. They are also aware of the needs these world-wide ventures have for computer programmers. One cannot therefore overlook the influence which such powerful concerns can exert on the choices pupils will make in the course of their studies.

Co-operation with neighbouring countries have had an impact on the mathematics curriculum bringing the change to the metric system, the introduction of International System of Units (S.I.) units and of certain topics in 'modern' mathematics, a new style of writing mathematics textbooks, common examinations, etc.

It is not possible to be exhaustive in such a short presentation, but it is important to note the strong influence of happenings elsewhere in the world. In 1957 the Russian success in launching the first Sputnik generated a lot of curricular activity in the developed countries; the consequent programmes, designed in those countries, had a great

impact upon the teaching of mathematics in the United Republic of Tanzania. The British School Mathematics Project (SMP) programme and the American School Mathematics Study Group (MSG) project played a significant role in changing the Tanzanian mathematical orientation. This will be discussed later. Suffice it to state here that no one single factor can be considered as solely responsible for what we teach and how we teach.

The teaching of mathematics in this country today reflects the demands of national development. These include needs of the civil service, the private business sector, the professions, and those of higher (tertiary) education. It would appear that utility is on top of the list, but the need to nurture the latent potential of the younger generations is also respected. The ideological stand of the political party, and the government policy on vocationally-oriented secondary education are other important factors today. In the area of ideology, the need to ensure equal access to education by all, and the emphasis upon the needs of the majority are paramount. The government policy on vocationally-oriented secondary education emphasizes the need to train youngsters who are employable immediately after graduation, or who will need very little additional training. At present, the discernable practical streams are agriculture, technology, commerce and home economics. These choices have important implications for mathematics teaching.

Modern Mathematics Projects

'Traditional' mathematics monopolized the scene in the United Republic of Tanzania at all levels until the mid-1960's. Then 'modern' mathematics was introduced. Previously, at the primary level, emphasis had been placed on the four basic operations of addition, subtraction, multiplication and division. Ratio, measurement, geometric drawing and simple accounting were also a part of the primary school syllabus. In Primary 1 to 4, the medium of instruction was Kiswahili, while, in Primary 5 to 8, the medium was English. Consequently attempts to produce mathematics textbooks in Kiswahili can be traced back to the early days of formal education. Those written for the upper classes in primary school took into account the local conditions, and were therefore easily understood by the pupils.

At the secondary level, British syllabuses were used by students preparing for British examinations. Textbooks written in the United Kingdom, by British authors for British schools, were imported and used in schools throughout the country. It is these books which reflected a foreign element, and which made life difficult for the

secondary school pupil. Alternative 'A' and 'B' syllabuses were on offer for the Ordinary level examinations, while, for Advanced level examinations, both 'single subject' and 'double subject' mathematics could be taken. As the medium of instruction at the secondary level was English (it still is), the syllabuses and textbooks were of course all in English. This feature, however, provided a common basis for co-operation in East Africa, and beyond in English-speaking Africa.

While scientific and technological changes were igniting curricular activity in the industrialized countries in the mid- and late-fifties, political ones were influencing the course of history in the colonies, e.g. the attaining of independence of the former Gold Coast, giving birth to Ghana. Chain reactions were sparked off whose impact is still felt today. The technological advances required greater changes at the school level in the form of revamped science and mathematics curriculum programmes. In the United States, several projects were started, among them the SMSG, the Greater Cleveland Mathematics Program (GCMP), the University of Illinois Committee on School Mathematics (UICSM) and so on. In the United Kingdom projects like the SMP, the Manchester Mathematics Group (MMG), the Scottish Mathematics Group (SMG) and the St. Dunstan's mark the preoccupations of the time. On the political scene, on the other hand, several political parties were formed, and those already in existence were revamped with a single mission, to deliver the colonies out of political bondage to political freedom.

At one point, the two influences converged. This happened when newly independent countries sought assistance from the developed ones. Leaders demanded quick solutions to their problems, and one of these was to resort to scientifically-based occupations. It was felt that by turning out a scientifically-orientated manpower, great strides would be made towards economic liberation just as there had been political liberation.

The most conspicuous event in East Africa was the meeting at Entebbe (Uganda) in June 1962 of several mathematicians and mathematics educators from English-speaking Africa and a few from other countries to produce textbooks in modern mathematics for use in English-speaking Africa. The motivating force was the United States. Consequently the products of the Entebbe workshop were books which reflected the new content of American programmes, especially that of SMSG. It will be pointed out later on in this study that the Entebbe workshop was the beginning of a big exercise which set in motion all sorts of activities aimed at modernizing not only the content of mathematics at the schools level, but also the whole approach to the teaching of mathematics in the curriculum.

The British were not to be beaten by the Americans. They, too, sought to influence the change in mathematics teaching in English-speaking Africa. Through the Centre for Curriculum Renewal and Educational Development Overseas (CREDO), the British Council and other bodies, they sponsored workshops in Nairobi to make the British SMP programme acceptable to Africa. One upshot was the launching of the School Mathematics Project for East Africa (SMPEA), which later changed its name to School Mathematics of East Africa (SMEA). Today one can still find traces of these two programmes. Others have taken a different turn and have evolved their own programmes, borrowing the best features from the different projects.

Other important factors have been responsible for introduction of modern mathematics in English-speaking Africa. One of these is the demands of those engaged in tertiary level education, particularly in the universities. Throughout the history of mathematics education at the school level, the influence of universities has been great. The United Republic of Tanzania is no exception. Very influential were members of the Mathematics Department and of the Institute of Education at the University College, Dar es Salaam. Without any exaggeration, they were the most influential group in the country in advocating the introduction of modern mathematics. They helped in drafting the syllabuses, later approved by Cambridge Examinations Syndicate. They helped to organize the annual refresher courses for mathematics teachers. They helped to write and to distribute mathematics materials. They organized and piloted the Mathematical Association of Tanzania, and promoted its activities – contests, competitions, etc. Most important of all was the key role they played through their training programme. It is through these tertiary teachers that the country was able to secure a core of well-trained maths teachers, well versed in modern mathematics.

An important influence was also exerted by a number of Tanzanians returning home from their studies abroad, especially in the United States. They had been exposed to the new ideas, and were eager to try them out in the schools. They, together with the expatriate staff who were aware of these new programmes, formed a useful nucleus in the schools. Through them, it was possible to influence other teachers who were not familiar with the new ideas. Through them, too, it was possible to organize zonal and regional workshops/seminars, where new texts could be studied, and help on certain topics given to their colleagues.

All these influences would, of course, have made no impact if it were not for the very warm reception the programmes received in Government circles. In countries where the direction of education is centralized, it makes a world of difference whether or not the centre is satisfied

about the advantages in and the need for change. Fortunately for the United Republic of Tanzania, there were men at the centre who were themselves mathematicians of the new type. They gave the project their full backing, and committed public funds to it so as to ensure its success. At a conference³ convened by the then Minister of Education in 1967, the Government made further commitments – the project was made one of the key issues in the attempt to implement the President's Paper 'Education for Self Reliance' in April 1967. No greater boost could have been received than this national commitment. Judging from some misgivings which emerged later on, it is not clear whether or not everyone knew what they were supporting. Perhaps they were supporting the principle rather than the whole package. But it could also be a reflection of the high expectations they had of the new programme. Failure to see tangible results after a few years has caused them to turn away from their original support, and there are cases where projects have been abandoned. There is need to discuss this development a little more later in this paper.

The diverse origins of the projects is reflected in the source of their funding. The Americans poured money in through the United States Agency for International Development (USAID) to support the series of workshops which started at Entebbe. This support made it possible to assemble experts in the field annually until the whole range of books and other supporting materials had been produced. They also supported the special seminars held in Nairobi to train a core of specialists, who would serve as resource people in their respective countries. When it was decided to hold annual refresher courses, the Americans supported the programme by paying for the experts from the United States and for the expenses incurred during the workshops. This went on until 1969, when the United Republic of Tanzania decided to be self-reliant and to meet all the costs themselves, with effect from the 1970 workshops.

The British also contributed their share to the project. As already stated earlier, they gave their support to the British-oriented SMPEA, which was developed at workshops organized in Nairobi. They also paid for the British experts who participated at the refresher course seminars organized annually alongside the Americans. British financial assistance came in other indirect ways also, as they provided curriculum facilities in general. The same can be said of the Americans who supported efforts in curriculum development through the provision of facilities and personnel.

3 Ministry of Education. *Conference on Implementation of Education for Self Reliance*, Dar es Salaam, United Republic of Tanzania, 1967.

Private foundations played their part through grants to the curriculum development centres. Such assistance, although not directly related to mathematics education, made it possible to organize the refresher courses already mentioned.

Perhaps the most important financial supporter was the Government. It was through the provision of public funds that it was possible to sustain the new mathematics programmes for so long. Funds were made available to produce texts, to get teachers to mathematics workshops, to train and re-train teachers, inspectors and other supporting staff. No figures are readily available to document this observation since these would normally appear under appropriate expenditure codes covering training or the production of teaching materials in general⁴. First-hand experience in running such courses and in producing textbooks for the schools indicates that governmental financial support was quite substantial.

During the seventies, Unesco and the United Nations Children's Fund (Unicef) came to the aid of the country with financial assistance for teachers courses, especially at the primary level. These courses covered several subjects, including mathematics. It would therefore appear that these agencies of the United Nations also played their part in supporting the programme financially.

Goals of Modern Mathematics Projects

It has already been pointed out that the United Republic of Tanzania has justified the provision of secondary and of higher education mainly to satisfy the manpower needs of the country. Mathematics, like other subjects, is therefore taught because it is an important part of the body of knowledge that must be imparted to those who will be trained as teachers, surveyors, engineers, scientists, etc. Because these professional fields are changing in ways which require a different mathematical background on the part of the student, it is inevitable that change must also come to the mathematics curriculum. For example, even in the poor countries of the world, the computer is already making an impact. This impact will be small to start with, but in the long run it is bound to snowball. Since some of the secondary school leavers will go into these fields, it is argued that modern maths, which provides an insight into the fundamentals of the computer, must be taught. Some of these school leavers will also need to use a slide rule, and must therefore learn how it works.

4 Mmari, G. *Ten Years of Curriculum Development in Tanzania 1964-1974*, East African Publishing House, Nairobi (In press).

Apart from those who have argued for modern maths along these lines, there are also those who see in 'new maths' some wider horizons. Instead of drawing graphs of functions which emphasize only equalities, they see the need to introduce inequalities also, as these are so important a feature of linear programming. Those who continue to study economics will find this skill very necessary. The study of statistics is very important in a world which continues to quantify relationships. We must therefore introduce statistics to as many pupils as possible. And so these reasons and others like them are adduced for introducing modern maths.

In the United Republic of Tanzania, the arguments for modern maths have been closely related to the basic philosophies of the Entebbe and SMEA projects. The former, which had been influenced by the American SMSG, has tended to emphasize the need to help the learner see the *why* as well as the *how* of mathematical processes. Rigour has been so much emphasized that the average and the weak have found the going a little rough. Research findings have shown that schools and even countries have dropped the project, mainly because they consider it too rigorous.⁵ It would therefore seem that a goal which emphasizes understanding through rigorous proof can have negative effects if it does not take into account the ability of the learners. Perhaps the teachers too are partly responsible where, not having mastered the material themselves, they have not been able to help their pupils.

The SMEA project, like that of Entebbe, introduced new topics, but with an emphasis on patterns. Mathematics, it is argued, is basically a study of patterns of relationships. The learner is therefore helped by the teacher to discover these patterns and see if he can draw intelligent general conclusions from them. The schools which adopted the SMEA programme felt that it was not too removed from the traditional one and had fewer problems than those who had adopted the Entebbe.

There are several critics who thought that the primary goal of the projects was to respond to the demands of the multinational corporations – IBM, book publishing firms, external examining bodies, etc. Their views were probably strengthened by the emphasis placed by the proponents of the projects on the uses of mathematics in a 'computer' age in a country where the hand hoe was still the main tool of the majority of the peasants.

Once the overall aims of modern mathematics had been defined, the national project set itself goals for implementing the programme. The

⁵ Mmari, G.P.V. *A Study of the Understanding of Mathematical Ideas and Concepts Among Tanzanian Secondary School Pupils*, Ph.D. dissertation, University of Dar es Salaam, 1973.

most important one was the writing of a teaching syllabus to go with the book which had already been developed at the Entebbe Workshop (later moved to Mombasa, Kenya). To do this, the Mathematics Panel of the Institute of Education set up a sub-committee to write the syllabus. The second goal was the training of teachers who would teach the syllabus using the books produced. This was an annual activity which drew about 100 teachers for a two-week seminar in Dar es Salaam. The third goal was to evaluate the effectiveness of the syllabus and the books used. Feedback from the exercise was to be used in rewriting and revising the books. Part of the evaluation exercise was to compare performance in modern maths with performance in traditional mathematics. The fourth goal was to spread modern mathematics to all secondary schools after evaluating performance in the experimental schools.

Today, there is a single mathematics programme in the United Republic of Tanzania.⁶ The Entebbe, SMEA and Traditional programmes have ceased to exist as separate mathematics programmes. This is the result of the careful study of the advantages and disadvantages of each *vis-a-vis* the needs of the country. The overall goal was to produce competent mathematics students basically equipped to pursue different vocations. It was found that there were certain key mathematical topics and concepts which all needed to learn. These were extracted from the different programmes to form the core of the Tanzanian mathematics programme. Although the original labels have disappeared, the philosophies have been retained. The revised core programme emphasizes understanding, active participation by the learner and practical use. Each teaching syllabus shows very conspicuously the accepted objectives of secondary mathematics education in the country. These are:

1. to develop mathematical skills among pupils which will enable them to function in all practical affairs of life;
2. to provide pupils with mathematical tools which they can use in other subjects;
3. to develop the abilities of pupils to discover mathematical concepts and ideas, and also their ability to think logically.

The decision to teach mathematics to every secondary school pupil for four years has had its repercussions. Not all students like mathematics, and are willing to exert themselves. Consequently the goal, though certainly an important one, is unattainable. There exist more advanced syllabuses for the more able, those who will specialize in

⁶ Ministry of National Education. Mathematical Subjects Forms I-VI. In: *Secondary School Science Syllabuses* Vol. 2. Ministry of National Education, Dar es Salaam, 1976.

mathematics or who will study subjects which require advanced mathematics. It might become necessary at a later date to reconsider, and redefine this goal in relation to the findings on performance so far.

Further discussion on goals as related to achievement will be discussed later. But this apparent observation on poor performance has far-reaching implications for the national goals for manpower development.

Development and Operation of the Project

The Tanzanian modern mathematics project can be traced to the 1962 workshop at Entebbe and subsequent annual workshops at Entebbe, and, later, at Mombasa. These workshops took place during the summer months so as to minimize any disruption in the institution from which the participants came.

The workshops succeeded in producing a primary series, complete with teachers' guides, and a secondary series at three levels: the general series for the average pupil; the 'additional' series for above average pupils in their third and fourth years at secondary school; and a third series for the pre-university, or 'Advanced Level' student (to use British terminology). The workshops also produced a series for teachers colleges. Some films were produced for training purposes, and a battery of tests for evaluation was also prepared.

Meanwhile, the SMP group was similarly engaged, except that its efforts were focused upon the secondary level. This group also relied on the British texts which had been written in the United Kingdom for British schools. Eventually the SMP group produced books for the first four years of secondary education, but no books for the other stages: primary and advanced level. For the latter, the British SMP books were used.

In 1973, it was decided that, after experiencing the trial materials, it was clear that a unified programme could be designed. The new programme selected the best features from the Entebbe course and the SMP course, or its East African derivation, the SMEA. The books resulting from this merger were simply called the Secondary Mathematics series. The process is not yet complete, and more changes, arising from experience, are anticipated as time goes on.

An important factor in planning for the project was personnel. This was tackled at two levels: the international and the national. At the international level, seminars were arranged in Nairobi. Participants were divided into groups depending partly upon their backgrounds, but more

importantly upon the needs of their respective countries.

The other approach to the programme involved the organization of seminars and workshops at the national level. Thousands of teachers have undergone retraining, including participants at the primary, secondary and teacher training levels.

In addition to these local efforts to develop the programme, other bodies organized and solicited help from the developed countries. Several experts participated in this project and their input and contributions went a long way towards achieving the goal set.

Communication

For the success of the project, it was necessary to ensure effective communication between the different groups of developers and the teachers in the school system. In the United Republic of Tanzania, this was not a problem since there is a centralized Ministry of National Education from whose head office several directives and guidelines were sent to schools and colleges. But, in addition to communication through the Ministry of National Education, other efforts were also made. For example, a newsletter was sent out by the mathematics curriculum developers. Through the newsletter, teachers in the experimental schools were able to keep in touch with their colleagues. It also made it possible to provide the schools with regular and useful guidance.

Another important aspect of communication was the founding of the Mathematical Association of Tanzania (MAT). It is strange that no efforts had been made prior to this period to start such an association. Zonal branches were established, and these brought together teachers of mathematics for the exchange of ideas and news. But central to their meetings was the project, and the meetings became very effective forums for solving common problems or for comparing notes or for producing teaching notes on difficult topics. These zonal branches reported to the committee of the Association which, in turn, circulated to other members important ideas emanating from the zones. At the end of every year, and coinciding with the annual workshop/seminar, an Annual General Meeting of the Association was convened, not just to elect office bearers and to receive the Treasurer's report, but also to review the progress of the project. Annual reports written for Annual General Meetings have proved very valuable sources of information.

In 1966, MAT established an official organ, the *MAT Bulletin*, which has come out regularly since its first issue. As may be expected, most of the articles were on modern mathematics. Some were on

content, some on methodology; others were book reviews, solutions to difficult problems, or comments on new efforts in the schools, etc. The strength of MAT and the *MAT Bulletin* lies in a core of very committed members in the Mathematics Department of the University of Dar es Salaam, Institute of Education (responsible for curriculum development) and teachers in the regions. An active branch has always revolved around a core of keen and active members.

One failure of communication has been in the education of the general public. Most of the recent criticism has come from the lack of publicity of both the successes and the difficulties of the new programme. This gap is to be expected in a developing country, where the rate of literacy is low, and the means of communication are limited. Recent criticisms can be explained partly by a general increase in the public awareness of what is happening in schools. More and more educated parents are able to follow what their children are learning at school. When they don't understand, they blame the teachers, the programme and the system.

The question of ideology has also figured in public debate, and questions on the usefulness of some topics for socialist transformation have been raised. These will be resolved with time as issues are thrashed out, discussed and clarified.

Decision-making

Decision-making has been fairly formalized and institutionalized. Figure 1 explains diagrammatically the lines of communication.

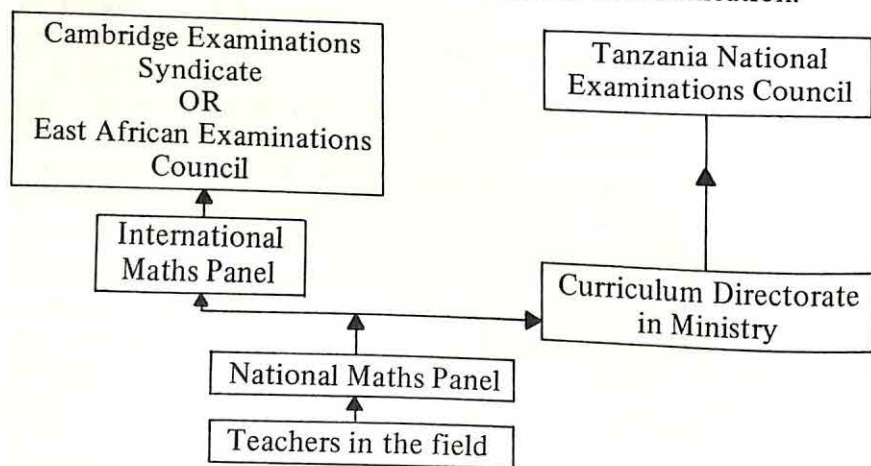


Figure 1. Decision-Making in Mathematics Curriculum Development

From the figure, two routes in the flow chart are shown. On the left is a route followed before the decision was taken in 1971 to set all examinations locally. On the right is the route followed today. Two differences between the two routes need emphasizing. The old route affected only the secondary school mathematics curriculum, whereas the new route affects the other levels as well. The second difference lies in the final control of the curriculum. Under the old arrangements, an external agent had the last word, either in the United Kingdom or in East Africa (in the East African Examinations Council). In the new, the final authority resides within the country, and is therefore internal. The internal authority is expected to be more responsive to the national aspirations and policies. In practice, this came out very clearly in decision-making on the future of the Entebbe and SMEA programmes. While the East African Examinations Council drew up syllabuses to accommodate the separate programmes, the Tanzanian decision was to draw up a national syllabus accommodating the essentials from all.

At the primary and teacher training levels, decisions on important policies, such as use of Kiswahili as a medium of instruction, have had to come from outside the structure. This was a political decision for the government to implement. But for all technical matters, the structure has generally been followed.

The learning requirement

The learning system has had to accommodate both the different needs of individuals as well as national requirements. At the national level, a policy statement in 1968 made mathematics a compulsory subject during the first four years of secondary education. This came as a blow to the below average who had expected to drop mathematics after the first two years. Nevertheless, the national demands for manpower in scientific and technical fields dictated the need for a universal background in mathematics for all secondary school leavers. So, in this particular matter, where the interests of the nation and of the individual were in conflict, those of the former prevailed. But the latter did not take it lying down. He paid back by ignoring mathematics lessons and scored nought in the final examinations. The general opinion among teachers of mathematics is that quite a number of pupils are capable of doing better. The first question is how to motivate them, and the second is what mathematics to teach them.

Due to the ideological orientation of education policies, streaming by ability is frowned upon. The resulting heterogenous classes are becoming difficult to handle, especially when differing abilities are compounded with a lack of motivation among some pupils after the

first two years. A partial solution for the better pupils lies in offering them the option of 'Additional' mathematics. This is bound to generate more homogeneous grouping, as classes consist mainly of above average and highly motivated students.

Teacher preparation

The preparation of teachers has already been discussed, but it would be useful to look at problems which have come up during the period of the project. The most irritating in the early days was the transfer of teachers regardless of the role they were playing in the experimental schools. When a common syllabus was laid down, this ceased to be a problem. Then there was the question of preparing the teachers. Despite all the efforts put into seminars, workshops, zonal meetings, etc., there were always some who found it difficult to handle some topics. In addition to this handicap, some teachers stuck to their old methods in teaching the new curriculum. The consequence was that they were in reality adding new topics to an old programme, thus defeating the spirit of the project.

After the trial period, and after the project had become a national one, new problems came to the surface. The poor preparation of some teachers became evident in bad teaching and poor performance. Not surprisingly, the teachers blamed this on the programme instead of themselves. Then there were difficulties with the textbooks and the teachers' guides. Some had been written in a hurry and had errors in them or were somewhat obscure. Again, some of the best teachers were transferred out of the system to administrative jobs, and this was a blow to the project. Officials of the Ministry responsible for education helped as best they could, but some were uncommitted, and this did not help matters. A teacher *needs support from his* superiors. When this is not forthcoming, his job becomes difficult enough without a lack of help with facilities – teaching aids, supplementary reference books – inadequate supplies of textbooks, delayed distribution of those available, etc.

The medium language

The language of instruction was also a problem, as it still is. Mathematics employs a pretty technical vocabulary. Yet ordinary language is essential for communication during teaching. The two must therefore be mastered if effective learning is to take place. After the 1967 decision to use Kiswahili as a medium of instruction, a host of problems has been created. Secondary school entrants do not have a good

command of English, and it takes time to acquire one. In this situation, two approaches to the problem have been taken. The first has been to provide a special language and study skills programme for all secondary school entrants.⁷ This was offered in 1968 and 1969, but, soon after, it was abandoned due to the extra cost of maintaining the pupils outside term time. Some schools have since integrated the programme into the school term, but the majority seem to have resigned themselves to the status quo. The second approach has been to mix the two languages during mathematics lessons. Official policy is opposed to this solution, but it is happening. The ultimate solution lies in the adoption of Kiswahili as the medium of instruction at all levels. The other alternative would have been to re-introduce English as the medium of instruction at the primary school level. This, however, is impossible for political reasons.

Efforts to develop Kiswahili so as to accommodate mathematical terms have so far been very successful.⁸ Some 631 terms have already been compiled, and, after another 70 terms have been approved, there will be a sufficient number of terms to handle mathematics up to Advanced (i.e. pre-university) level. The next issue will be the medium of instruction at university level.

Content and method

The mathematical concepts included in the programme are closely related to the experience of those who took part in designing the programmes of the early sixties. As already pointed out, the Entebbe programme had been strongly influenced by the American SMSG, and the SMEA by the British SMP. Serious discussion on which, if any, of the newer concepts should be retained took place in the early seventies, when teachers were invited to suggest what was good for the nation and the pupils. As a result, a number of topics was retained from the earlier syllabuses, among them: sets, number and numeration systems, matrices, vectors, equations and inequalities, relations and functions, slide rules and tables, transformation, enlargement, statistics, probability.

Because of the emphasis of the country's philosophy upon utility, the following topics were introduced: simple book-keeping, plan and elevation, kinematics, rate, ratio and proportion. Traditional topics were also retained. These include sequences and series, approximation

⁷ Government of the United Republic of Tanzania. *The Second Five Year Development Plan 1969-74*, Dar es Salaam, 1969 (discusses the policy implications).

⁸ Unpublished papers at the Institute of Education, Dar es Salaam; the Ministry of National Education; and by some Mathematics Educators.

and accuracy, geometry, co-ordinate geometry, identities, circle and sphere, trigonometry.

There was also less insistence upon rigour for the average pupil, but the discovery approach was retained. An emphasis was placed upon project work wherever this was practicable. Examples were given in the syllabus.

The ultimate choice of topics depended upon a variety of influences: teachers in the schools, examination results, national needs, the length of the previous syllabus, and an overall assessment of the different previous programmes.

The unifying concept of 'sets' has remained, and is emphasized. Topics are developed in a spiral manner, so that general concepts repeatedly recur, but a greater depth each time and in a more abstract form. For example, Form I pupils begin with natural numbers and rational numbers, while real and complex numbers are taught much later. This approach is in contrast to the block approach, which tries to cover all aspects of a topic before a new one is tackled. The approach is also a contrast to programmes in some countries which offer a year of algebra, a year of geometry, a year of calculus, etc.

Evaluation and Achievement

The original project had built into it an evaluation component. A battery of tests was designed at Mombasa in 1965 and tried out in a number of experimental schools. A 1966 report showed that each programme had its strong points. The good performance in some modern topics was attributed to superior teaching and the Hawthorne effect. But the generally low overall results were of concern to the project organisers.

Since the 1965 evaluation, there has been no comparable testing programme at the secondary level. Reliable indicators have, however, been provided by examination results, especially those in examinations set by the Tanzanian National Examinations Council. These were particularly valuable because they provided feedback to the teachers almost immediately, whereas the Cambridge Syndicate provided detailed feedback after a lapse of about two years.

Another evaluation procedure was the calling for chapter by chapter reports from schools using the modern mathematics materials. Carefully worked out questionnaires were sent to each school taking part in the project. These were completed and returned to the project coordinators. Their coverage invited opinion on content, depth, adequacy of exercises, performance by pupils and suggestions for improvement.

Teachers were also asked to point out printing errors and other mistakes. Those who faithfully filled and return the forms made it possible for some very definite conclusions to be drawn. The feedback was also used in rewriting the materials and in improving the programme.

Dividends

The effect of the project on the Tanzanian educational system has been enormous. It provided a unique opportunity to mathematics teachers to design their own curriculum. Previously, they had passively bought their copies of the prescribed syllabuses and the relevant textbooks in the bookshops. The project, in contrast, released their creative energies. Some participated in writing the textbooks; others, the teachers' guides. Some contributed by criticizing the work of others. For all, it was a wonderful experience. It gave teachers an opportunity to relate universal mathematical principles to the Tanzanian reality. In such writing workshops or learning seminars, a spirit of comradeship was forged. They were united in a single purpose – to produce their own teaching materials.

It can also be said without any exaggeration that the project also provided a catalyst for the growth of professionalism in mathematics education. A Mathematical Association was started which has now survived for over twelve years. It has an active programme spreading through all the different levels, as has already been described. The Association, in short, has become a forum for raising the quality of mathematics teaching.

An important side effect has been the great enthusiasm to evolve the Kiswahili mathematics vocabulary. This drive has been aided by the political climate which favours bilingualism, but with a slant in favour of the *lingua franca* – Kiswahili. This enthusiasm is unlikely to die down even if the decision to switch to Kiswahili as the medium of instruction at the secondary school level is not made soon.

The opportunity for retraining teachers is implied. The older teachers have learnt something even if they are not all fully committed to the new programme. Their participation in seminars, workshops, etc. has helped expose them to new ideas, and to the strong winds of change.

The project has not been a total success. Some reasons for this have already been cited. They include the mixed background of the teachers (by training, age and nationality), the inadequacies of provision of facilities at the national level, the political changes (especially the policy decision on the language of instruction at the primary level and its effect at the secondary), the breakup of the East African Exam-

inations Council (and thus loss of inputs from Kenya and Uganda), a critical public (who do not see the point of making a change), the ideological debate (on the real motive behind the change) and some other related problems.

Material products

The materials available at the secondary level now include a complete set of textbooks (see figure 2).

Figure 2

Book	Level
Secondary Mathematics 1 – 4	Ordinary Level
Teachers' Guides 1 – 4	Ordinary Level
Additional Mathematics 1 – 2	Ordinary Level
Teachers' Guides 1 – 2	Ordinary Level
Advanced Mathematics 1 – 2	Advanced Level
Teachers' Guides 1 – 2	Advanced Level

All these books were produced and printed locally. The teachers' guides for Secondary Mathematics Books 2 – 4 were produced cheaply by cyclostyling gestafax stencils. The binding is good and strong. The number produced is much smaller than the printed books.

Other publications include the printed syllabuses listed in figure 3.

Figure 3

Syllabus	Level
Basic Mathematics	Ordinary Level
Additional Mathematics	Ordinary Level
Basic Applied Mathematics	Advanced Level (Subsidiary)
Advanced Mathematics	Advanced Level

These are available in bookshops as well as from the Ministry of National Education.

In addition to the books and the syllabuses are the regular issues of the *MAT Bulletin*, the Mathematics Seminar Reports and a new addition, the *MAT Junior Bulletin*, introduced to meet the needs of the school pupils. Some Kiswahili titles have also appeared, but mainly at the primary level and for use in teacher training institutions. In pre-

paration for use at the secondary level is the mathematical vocabulary list in Kiswahili.

Conclusions

An attempt has been made to sketch in a rather brief fashion the project which ushered in the 'modern' or 'new' ideas and approaches to mathematics education in the United Republic of Tanzania. It has been shown that the project has not suffered the extreme fate of similar projects in other countries. That it has survived the tempest is due to various factors. But it has also been shown that internal pressures have made it necessary to effect a compromise by allowing modifications. These pressures have come from the immediate consumers, namely the pupils, from the executive agents, namely the teachers, from the assessors, namely those responsible for public examinations, and from needs of society as expressed by the political and government leaders.

The United Republic of Tanzania is not and cannot be an isolate. It must interact with the rest of the world. New ideas in mathematics and in mathematics education will flow in bringing change in their wake.

One of the most important changes will occur when the nation finally settles the question of the medium of instruction. If, eventually, Kiswahili becomes the language of instruction at the secondary level, then a major drive will have to be made to accommodate the change. All textbooks will have to be translated into Kiswahili. Teachers will have to attend courses in Kiswahili in order to learn how to handle the technical terms coined for the purpose. This will be especially difficult for expatriate teachers whose command of ordinary Kiswahili is minimal.

As already pointed out, the work of building the necessary vocabulary has already begun. These efforts will make the transition less difficult. It is assumed, however, that the country will remain bilingual in respect of its official languages. Reference books written in English will therefore become accessible to pupils and teachers. This is already the case in a number of countries which have decided to adopt their national languages while retaining a metropolitan language alongside.

The third challenge will remain, that is the need to give the mathematics curriculum a practical slant. The national philosophy of education stresses the importance of the practical application of knowledge. It is obviously impossible to exclude all topics in mathematics which do not lend themselves to application. Many topics and concepts are a necessary means to the understanding of the 'applied' ones. Not only that. As long as man's nature remains what it is, he will always strive

to know more, even if it is not evidently of immediate use. The Tanzanian man is no exception.

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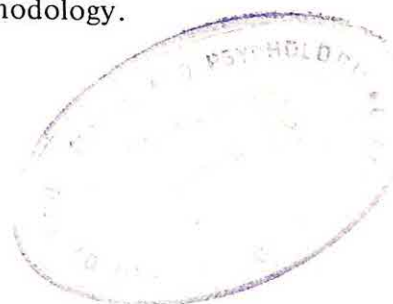
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